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# PHYSICAL REVIEW.

## ON THE DIFFERENTIAL TRANSFORMER.

By A. Trowbridge.

THE principle of the differentially wound current detecting instrument, of which the differential galvanometer is the type, has been adopted by H. Ho,[1] and later by Duane and Lory,[2] for the comparison of inductances and capacities by means of alternating currents.

The instrument described by each of these authors is a telephone the bobbin of which is wound in twin in such a way that, when equal and oppositely directed currents are sent through the coils constituting the twin winding, no magnetic field which reaches as far as the diaphragm of the telephone is set up. This we may call the first condition which the instrument must fulfil.

The second condition is that the coefficients of self-induction of the two coils must be equal.

Both conditions are fulfilled if the two coefficients of self-induction are equal to each other and either is equal to the coefficient of mutual induction between the two circuits, since in this case no magnetic field whatever would be set up if equal currents were to flow in opposite direction in the two circuits. If, moreover, the ohmic resistances of the circuits are equal, or are made equal by the addition of suitable non-inductive resistance in series with the coils, then when an alternating electromotive force is applied to the two circuits

[1] Electrical World and Engineer, May, 1903.

[2] Phys. Rev., Vol. XCVII., p. 275, 1904.

in parallel the currents are the same in each and the telephone behaves as a perfectly non-inductive resistance and gives out no sound even when carrying relatively strong currents through its two coils.

Thus an instrument which fulfills this last double condition relative to its three coefficients of induction becomes an exceedingly sensitive device to detect whether or not two alternating currents are equal in amplitude and are in phase with each other, for only in the case of equality both of amplitude and phase is the instrument silent.

H. Ho was able, it seems, to construct a telephone which fulfilled the condition of equality between the three coefficients of induction of the twin coils and for this reason might be treated as perfectly inductionless, thus making it applicable to a great variety of uses, but the accuracy he reported was very much less than that obtained by Duane and Lory, who constructed a telephone for which the condition of equality of the three coefficients of induction was only approximately fulfilled.

These authors, finding great difficulty in winding the twin coils so that they might be exactly equivalent in their magnetic effect on the diaphragm, placed in series with one of the coils a small movable loop consisting of a few turns of wire, and by adjusting the position of this loop with respect to the diaphragm were able to bring the telephone to silence when the same current flowed through the coils in opposite directions.

In this manner the first condition was fulfilled, but it was found that the two coefficients of self-induction were not equal and, of course, the coefficient of mutual induction differed slightly from either one of these.

By adding a small adjustable inductance in series with each of the twin coils, the self-inductance of the two systems thus formed could be brought to equality, and so an alternating electromotive force applied to the two systems in parallel would send equal currents through each, provided the resistances were adjusted to be equal, and these two currents would be in phase with each other, though not in phase with the applied electromotive force.

The fact that the inductances of the systems are not zero when in the balance condition somewhat restricts the usefulness of the instru-

ment, but Duane and Lory reported a sensibility of $\frac{1}{50}$ per cent. in the comparison of self-inductions, while H. Ho only reported 1 per cent. However, the latter states in a subsequent paper that he attributes the higher degree of accuracy obtained by Duane and Lory to their experimental skill and not to their method of applying the differential principle, and justly points out that his method of winding the coils is superior to theirs, since in his instrument the coils were non-inductive in the state of balance.

Having used the Duane and Lory type of differential telephone for some time and having found it a remarkably sensitive device and one very easy to use, it seemed well worth while to attempt to obviate what appeared to be its defects, preserving at the same time its great sensibility.

The most serious defect appeared to me to be the inductance of the coils in the state of balance and this I have been able to overcome, while still maintaining an accuracy at least equal to that obtained by Duane and Lory, and thus about fifty to a hundred times that reported by H. Ho.

It seemed to me a mistake to attempt to make the instrument portable, for in raising it to the ear and lowering it again, it is liable to get out of adjustment and further, unless the lead wires are perfectly non-inductively wound, their position relative to the diaphragm influences the balance point and thus an irregular source of error is introduced. A better plan seemed to be to wind a Faraday ring with a double primary in twin and a fine wire secondary to connect, by means of non-inductive lead wires, to an ordinary telephone. In this manner all current carrying lead wires could be fixed once for all and the essential part of the apparatus could be made larger, and therefore more easy to adjust without at the same time making the instrument too bulky. Figure 1 shows the connections including a slide wire and convenient switch board device for inserting in the circuits the inductances, capacities, or combinations of inductances and capacities to be compared, and one which permits one or both of the twin coils to be shunted with non-inductive resistance. In the figure $AB'$ represents the primary consisting of the twin coils $AB$ and $A'B'$ which are wound on an iron core and consist of about 1,000 turns each of No. 24 copper

wire — with a little care in winding, their coefficients of self-induction may be brought equal to within one part in 20,000.

The secondary $S$ connects with an ordinary telephone. The iron of the transformer core need not be soft nor is it necessary to laminate it, since at the balance there are no eddy current or hysteresis losses.

After setting up the instrument an alternating e.m.f. is applied to be the terminals 1 and 2 and all the plugs except $d$, $f$ and $e'$ are removed from the switch board; this arranges the connections so that the same alternating current flows through the two coils in the opposite direction.[1] If properly wound no sound should be heard in the telephone even when a current of a couple of amperes is flowing. In one of the instruments I have made there was a slight inequality in magnetic effect of the two coils due to carelessness in winding, but the inequality could be compensated by a single loop of wire in series with either coil brought to within three or four centimeters of the secondary. By twisting this loop between the fingers the sound could be made to cease in the telephone, and, on further twisting in the same direction, to begin again.

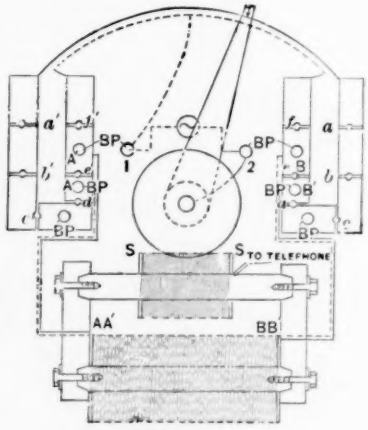

Fig. 1.

After testing for equal magnetic effect plugs $e$ and $e'$ are removed from the switch board, all the others being in place, and the slide wire is moved until the telephone is silent. With careful winding it should be possible to get a complete extinction of the sound even when the current through each coil is as great as two amperes. At any rate, a very sharp minimum is always obtainable even with rather careless winding.

If the instrument is found to satisfy the conditions both of equal-

[1] Resistance coils are inserted in the openings $a$, $b$, $c$, $a'$, $b'$ and $c'$ so that the equivalent length of the slide wire may be varied. When the plugs are removed from the openings $d$, $f$, $d'$ and $f'$ the circuit is broken at these points.

ity of magnetic effect and equality of inductance, it is ready for use in the comparison of inductances, either by the multiple arrangement of connections described by Duane and Lory, or by the series arrangement described by H. Ho. I will not enter into the details of these methods as they may be obtained from the original papers, but by way of an example to show the degree of accuracy attainable with the differential transformer, I will give the results of a series of observations taken by me in the comparison of a fixed inductance of about 25 milli-henries with a variable standard manufactured by Leeds and Northrop. The series was obtained adopting the multiple arrangement of connection in which the variable standard was placed in series with one of the twin coils and the fixed inductance in series with the other coil.[1] In order to increase the accuracy in the setting of the standard, a mirror was placed horizontally on the prolongation of the vertical axis of the standard and the setting was observed by means of a telescope and scale placed about ten feet from the instrument — a string passing over a pulley on the axis of the movable coil of the variable standard made it possible to vary gradually the amount of inductance of the standard until the sound was extinguished in the telephone.

Having previously determined that 1 degree on the scale of the variable standard corresponded to 13 cm. displacement of the scale of the optical pointer, I made twenty consecutive settings for balance when the variable standard was in series with the coil $A'B'$ and the unknown in series with the coil $AB$. In this case the greatest departure from the mean was 1.4 cm. or about one tenth of a degree of the scale of the standard, the setting corresponding to the mean being 85.17° — that is, the greatest departure from the mean was in the neighborhood of one tenth per cent. The mean of a similar set of observations with the variable standard in series with $AB$ and the unknown in series with $A'B'$ was 85.16° which shows that the twin coils were symmetrical to within one part in 8,500, or to within one eighty-fifth of one per cent.

Another double series of ten observations taken when the laboratory was free from disturbing sounds gave practically the same mean value of the unknown inductance, and a maximum departure

[1] In the openings $d$ and $d'$ respectively.

of any single observation from the mean of only one fortieth per cent.

It is thus very easy to compare inductances by means of the differential transformer to an accuracy of one tenth per cent. and with a little care an accuracy of a few hundredths of a per cent. may be attained.

H. Ho has pointed out that if the differential measuring instrument satisfies the condition of equality between its coefficients of induction, it may be employed to advantage in the comparison of capacities, since the twin coil in series with the greater capacity may be shunted with a non-inductive resistance, and the law of division of currents which holds for non-inductive circuits may be rigidly applied to obtain an expression for the ratio of the two capacities when silence in the telephone indicates equality of current and phase in the two twin coils.

The following data were obtained with the differential transformer in the comparison of two subdivided mica condensers, having a range of from 0.05 to 1.00 microfarads. Both condensers were made by Elliott, the one called $C_1$ having been in use four years, and that called $C_2$ being a new instrument with a capacity nearly 2 per cent. greater than the other.

The ratio of the capacities was found to be $C_2/C_1 = 1.01735$ when $C_2$ was in series with the twin coil $AB$ which had, therefore, to be shunted with 860 ohms, as its resistance was 14.91 ohms. On exchanging the positions of $C_1$ and $C_2$, the ratio was found to be

$$\frac{C_2}{C_1} = 1.01745.$$

A separate determination taken some weeks later gave values 1.01765 and 1.01785. These values show that the assymetry of the transformer coils is not greater than .02 per cent. while the mean of the two separate sets of observations agree to within about .03 per cent. The ratio of the same capacities determined by Gott's method was 1.018, the degree of accuracy obtainable being only about one tenth per cent.

As a check on the value obtained for ratio $C_2/C_1$, a separate series of observations was made on the ratios of the component

parts of $C_2$ and $C_1$, and from the data obtained the ratio between the capacities $C_2$ and $C_1$ was calculated and found to differ from that directly observed by less than one part in ten thousand.

When we consider that the differential transformer was constructed of low resistance so that large currents might be employed in the comparison of inductances, these results obtained in the comparison of capacities are remarkably good. It should be easy to increase the accuracy in the comparison of capacities by using a transformer the twin coils of which consisted of a great many turns of fine wire.

The condition of resonance either in a series or parallel arrangement of capacity and inductance is determined not only by the values of the capacity, inductance and resistance, but also by the frequency of the alternating current. In employing a telephone to detect this resonance condition, it is advisable to adopt some scheme of connections which will make the resonance condition practically independent of the frequency, especially if the alternating electromotive force employed is not a simple sine function of the time, since otherwise perfect silence at the balance cannot be secured owing to the presence of harmonics in the current.

The following scheme of connections for use with the differential transformer (or telephone) makes it possible to offset the lead due to the capacity $C$ by the lag due to the inductance $L$ so that the current will flow in phase with the impressed electromotive force (as through a non-inductive resistance), and in addition the balance condition is sensibly independent of the frequency.

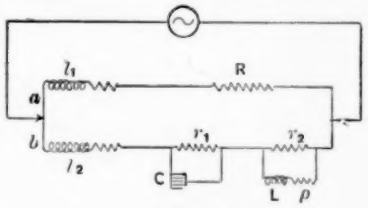


Fig. 2.

This desirable end is accomplished by a slight loss in sensibility which seems to be amply compensated for by the resulting agreeable sharpness of the balance.

In Fig. 2, $A$ represents the source of alternating electro-motive force, $l_1$ and $l_2$, the twin coils of the differential transformer, $a$ and $b$ the portions of the variable resistance which may be put in series with the coils $l_1$ and $l_2$ respectively, $R$ a non-inductive resistance in series with the coil $l_1$, $r_1$ and $r_2$ other variable non-inductance resist-

ances which serve as shunts to the capacity $C$, and to a variable inductance $L$ of constant resistance $\rho$ respectively. The twin coils $l_1$ and $l_2$ are here shown separate but in point of fact are superimposed. The secondary coil of the differential transformer and its connections to the telephone are not shown in the figure.

The complex expression for the admittance of the capacity $C$ shunted by the resistance $r_1$ is

$$\frac{1}{Z} = \frac{1}{r_1} + \frac{1}{jc} \tag{1}$$

where $c$ = the capacitance $= 1/C\omega$ and $j = \sqrt{-1}$. The expression for the admittance of the variable inductance shunted by the non-inductive resistance $r_2$ is

$$\frac{1}{Z'} = \frac{1}{r_2} + \frac{1}{\rho - jx} \tag{2}$$

where $x = L\omega$. From these expressions the impedances may be calculated

$$Z = \frac{r_1 c^2}{r_1^2 + c^2} + j\frac{r_1^2 c}{r_1^2 + c^2} \tag{3}$$

$$Z' = \frac{\rho(r_2 + \rho)r_2 + r_2 x^2}{(r_2 + \rho)^2 + x^2} - j\frac{r_2 x^2}{(r_2 + \rho)^2 + x^2}. \tag{4}$$

It is not necessary to complicate the problem by the introduction of the inductances of the twin coils, since in the state of balance each behaves as a non-inductive resistance.

In order that the current through the circuit containing the coil $l_2$ may be in phase with the impressed electromotive force, and hence in phase with the current through the coil $l_1$, the coefficients of the $j$ components must be equal

$$\frac{r_2^2 x}{(r_2 + \rho)^2 + x^2} = \frac{r_1^2 c}{r_1^2 + c^2}. \tag{5}$$

This is a necessary but not a sufficient condition for silence in the telephone.

The other necessary condition is that the currents in the two coils $l_1$ and $l_2$ shall be equal in magnitude at every instant. That is, the real components of $Z$ and $Z'$ added to the ohmic resistance

of the coil $l_2$ and the resistance $b$ in series with it shall equal the resistance $R + l_1 + a$.

$$\frac{r_1 c^2}{r_1^2 + c^2} + \frac{\rho(r_2 + \rho)r_2 + r_2 x^2}{(r_2 + \rho)^2 + x^2} = a + l_1 + R - l_2 - b = R'. \quad (6)$$

From these two conditions it follows that

$$x = (r_2 + \rho)\frac{r_1^2 c}{c^2 r_1 + (r_2 - R')(r_1^2 + c^2)} \quad (7)$$

or

$$L = (r_2 + \rho)\frac{r_1^2 C}{r_1 + (r_2 - R')(r_1^2 C^2 \omega^2 + 1)}. \quad (8)$$

The term $r_1^2 C^2 \omega^2 + 1$ which appears as a factor of a part of the denominator does not differ in practice much from unity, for, assigning the most unfavorable values to $r_1$, $C$ and $\omega$, say 200 ohms, 1 microfarad and $120\pi$ respectively, the term in question is 1.006. Even in the most unfavorable case, an error of .6 per cent. in the value of the term $(r_2 - R')$ would be introduced by neglecting $r_1^2 C^2 \omega^2$ with respect to unity. Now the term $(r_2 - R')$ is in practice always small with respect to $r_1$. In the most unfavorable case

$$\frac{r_2 - R'}{r_1} = \frac{1}{50} \quad (9)$$

so that the error in the value of $L$ introduced by neglecting $r_1^2 C^2 \omega$ is always extremely small, and we may write

$$L = \frac{(r_2 + \rho)r_1^2 C}{r_1 + r_2 - R'}. \quad (10)$$

Now since, to a close order of approximation,

$$R' = r_1 + \frac{r_2 \rho}{r_2 + \rho}, \quad (11)$$

we have

$$L = \frac{(r_2 + \rho)^2 r_1^2 C}{r_2^2} \quad (12)$$

and since $r_1$, $r_2$ and $\rho$ may be accurately determined, this permits a calculation of $L$ in terms of $C$.

It is evident that the smaller $(r_2 - R')$ is the less will be the disturbing effect of the term containing $\omega$.

If $\rho$ is small compared to $r_1$ and $r_2$ then $R'$ is about equal to $r_1$ and thus $r_2$ should be about equal to $r_1$ for the greatest sharpness of balance.

Using the differential transformer described in this paper, I was able to compare inductances ranging from 7 to 40 millihenries with capacities ranging from 0.05 to 1.00 microfarad to an accuracy of a little better than one per cent. for a single observation.

The reason the sensibility is so low compared to that reported in the case of the comparison of inductances or of capacities is due to the fact that the differential transformer used was designed for use with relatively large currents, while the method of comparison of inductance and capacity necessitates the use of relatively high resistances $r_1$, $r_2$ and $R$, and hence, of weak currents.

I do not report any series of observations at this time because I intend to repeat the experiment with the aid of a transformer especially designed for the purpose, since I am confident that an accuracy may be attained in the comparison of inductance and capacity by this method as great as that which I have reported for the comparison of inductances.

I base this statement on a series of observations, which my colleague Mr. A. H. Taylor very kindly secured for me, adopting a Wheatstone bridge and sensitive telephone. Three arms of the bridge consisted of non-inductive resistance, and the fourth consisted of the combination of resistance, inductance and capacity which is shown in Fig. 2 in series with the twin coil $l_2$. An alternating electromotive force of 110 volts was used and a complete extinction of sound could be attained on properly adjusting the values of $C$, $L$, $r_1$, $r_2$ and $R$. With 500 ohms in each of the proportional arms of the bridge a series of ten observations showed a maximum departure from the mean of one twenty-fifth of one per cent.; with 200 ohms in each of the proportional arms a maximum departure of one forty-second of one per cent. No disturbing effect, due to the complex wave form of the alternating electromotive force used, was noticed. This fact is, of course, explained by the absence of $\omega$ from the equation expressing the condition of balance which for the

bridge arrangement is substantially the same as for the differential transformer. The values of the inductance obtained by this bridge method were in accord with those obtained by the differential transformer to within the limit of error of the latter instrument. The limit of error of the bridge method, which is sufficiently small for most purposes, can, I am sure, be equaled by employing a properly designed differential transformer.

The question naturally arises as to whether or not the resistance $r_2$ (Fig. 2) could not be suppressed and the circuit still retain its property of being non-reactive for all frequencies. Treating the problem in a manner similar to that given above and employing the same symbols, the equations

$$L = \frac{r_1^2 C}{1 + C^2\omega^2 r_1^2} \tag{13}$$

and

$$\rho + \frac{\dfrac{r_1}{C^2\omega^2}}{r_1^2 + \dfrac{1}{C^2\omega^2}} = R' \tag{14}$$

may be deduced.

The first of these shows that the term $C^2\omega^2 r_1^2$ may not be neglected with respect to unity with anything like the same degree of accuracy as in the other case, where $1 + C^2\omega^2 r_1^2$ was multiplied by a very small quantity $(r_2 - R')$. Other things being equal, this would result in a tone minimum at the balance instead of practically silence as in the other case.

The second of the above equations shows that the resistance $R'$, required to balance $R$ is less than $r_1 + \rho$, which is the value $R'$ would have if steady currents and a differential galvanometer were employed instead of the variable currents and differential transformer.

This diminution in the effective resistance of the circuit amounts to 1.3 ohms (or about one half of one per cent.) if we assume the same unfavorable values of $r_1$, $C$ and $\omega$ mentioned above.

If the resistance $r_2$ is not suppressed the effective resistance differs from the true resistance by a much smaller quantity. This is shown when we form the difference, $\Delta R'$, between $R'$ as given in equation

6 and $r_1 + r_2\rho/(r_2 + \rho)$, which is the true resistance of the portion of the circuit in question.

$$\Delta R' = \frac{r_1^3 C^2\omega^2}{r_1^2 C^2\omega^2 + 1} - \frac{r_2^2 L^2\omega^2}{(r^2 + \rho)\left((r_2 + \rho)^2 + L^2\omega^2\right)}. \qquad (15)$$

A close approximation of this value may be obtained if we substitute the value of $L$ give by equation 12 and assume $r_1$ equal to $r_2$ which is the condition for maximum sensibility

Thus
$$\Delta R' = \frac{r_1^3 C^2\omega^2}{r_1^2 C^2\omega^2 + 1} - \frac{r_1^2(r_1 + \rho)C^2\omega^2}{1 - (r_1 + \rho)^2 C^2\omega^2}. \qquad (16)$$

Since $r_1^2 C^2\omega^2$ is always small with respect to unity it is evident that $\Delta R'$ tends toward zero with decreasing value of $\rho$. In the experiments described $\rho$ was ten ohms and so, if we again assign the same unfavorable values to $r_1$, $C$ and $\omega$ we obtain $\Delta R' = 0.07$ ohm. or about $\frac{3}{1000}$ of one per cent.

It is thus evident that the function of the resistance $r_2$ is to make both $L$ and $R'$ practically independent of the frequency of the alternating current employed and this result is well confirmed by experiment as it was never possible to obtain silence in the telephone when the resistance $r_2$ was suppressed.

PHYSICAL LABORATORY, UNIVERSITY OF WISCONSIN,
November, 1904.

# THE HEAT OF VAPORIZATION OF LIQUID AIR.

By R. C. Fenner and F. K. Richtmyer.

ATMOSPHERIC air ordinarily contains 20.8 per cent. oxygen by volume, about .63 per cent. argon and from 4 to 8 parts in 10,000 carbon dioxide, the rest being chiefly nitrogen. In the liquefaction of air, all constituents except the oxygen and nitrogen are removed as far as possible. Liquid air, therefore, is principally a mixture of the two liquids.

Liquid nitrogen under atmospheric pressure boils at 77.5° C. and liquid oxygen at 90.6° absolute. Liquid air would therefore have some intermediate temperature, which would mean that the nitrogen was superheated and the oxygen undercooled. Nitrogen therefore would boil off first, and we should expect as the liquid evaporates, its composition would change, and the liquid become richer in oxygen.

This relation between percentage composition and total evaporation of both the vapor and liquid phases has been investigated by several observers using different methods, and it is interesting to note that the curves thus obtained by different observers do not agree. It was noticed while experimenting in the present case, that when a liquid rich in oxygen was desired, slow evaporation produced the best result. The rate of evaporation will alter the percentage composition of both the vapor and liquid, and it is very probable there would be a set of curves for each rate of evaporation and that the differences in the curves of the different observers are thus to be explained.

As the percentage composition of the liquid is changing, varying from a liquid rich in nitrogen to one rich in oxygen, there would be a relative temperature change in the liquid. The curves showing the temperature with the vapor and liquid compositions are given in Fig. 1, and the data taken from Baly,[1] in Table I.

[1] Baly, Phil. Mag., 49, 217.

TABLE I.

| Absolute Temp. | Per Cent. O. in Vapor. | Per Cent. O. in Liquid. | Absolute Temp. | Per Cent. O. in Vapor. | Per Cent. O. in Liquid. |
|---|---|---|---|---|---|
| 80.7 | 16.8 | 44.3 | 83.5 | 33.3 | 66.6 |
| 81.0 | 17.5 | 47.3 | 85.1 | 44.6 | 73.8 |
| 81.3 | 19.5 | 51.0 | 86.7 | 58.0 | 81.5 |
| 81.7 | 22.2 | 55.1 | 88.8 | 77.0 | 91.0 |
| 82.4 | 26.1 | 61.5 | 90.2 | 91.9 | 96.2 |

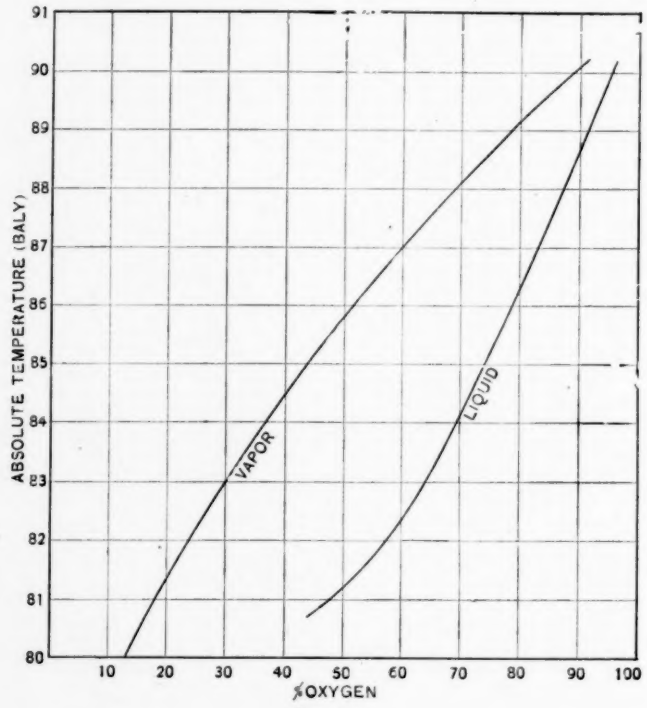


Fig. 1.

For the determination of the heat of vaporization in the present experiment, the source of heat supply was an electric current; the arrangement of apparatus and method of determination was precisely that used by Professor J. S. Shearer in his determinations.[1] The heating current was adjusted before each set of observations, so that when the heating coil was immersed in the liquid, the heat supplied was approximately one calorie per second. Higher values than this resulted in very irregular and explosive evaporization as

[1] Shearer, PHYSICAL REVIEW, September, 1902.

though the nitrogen had become superheated and then suddenly vaporized. When no current was passing in the heating coil, the rate of evaporation observed was due to conduction, etc., and thus this heat supply could be eliminated from the heat supplied by the current.

All the values here tabulated with the exception of the last two were determined under most favorable conditions. The high percentages of oxygen were obtained by very slow evaporation of the air. The air was analyzed by pyrogallol absorption and great care was taken to keep safely within the limit of its analytical absorbing power which is from two to two and one-quarter times its own volume.

The values for the heat of vaporization of liquid air as obtained by the method just described were quite unexpected. Thermodynamic computation as well as direct observation gives the heat of vaporization of liquid oxygen as about 61.0 calories per gram; and that of liquid nitrogen about 49.2 calories per gram.[1] One might

March 4, 1904. Bar. { 75.30 cm. / 75.40 " Room temp. { 17.40° C. / 19.25° C.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Voltmeter about standard coil. . . . . | | .685 | .685 | .685 | .685 | .685 | .685 |
| Voltmeter about heating coil. . . . . | | 6.18 | 6.18 | 6.19 | 6.18 | 6.18 | 6.18 |
| Temperature of entering gas | C.[2] | 18.07° | 18.10° | 18.50° | 18.82° | 18.90° | 19.30° |
| | No C.[3] | 18.07° | 18.26° | 18.68° | 18.80° | 19.18° | 19.25° |
| Temperature of gas leaving. | C. | 18.25° | 18.27° | 18.67° | 18.98° | 19.13° | 19.38° |
| | No C. | 18.25° | 18.43° | 18.83° | 18.94° | 19.40° | 19.31° |
| Volume in liters. . . . . | C. | 15.0 | 15.0 | 15.0 | 12.00 | 12.00 | 10.00 |
| | No C. | 1.50 | 2.00 | 1.00 | 1.25 | 1.00 | 1.00 |
| Per cent. O . . . . . | | 85.4 | 87.0 | 89.20 | 90.80 | 91.80 | 93.0 |
| Time in sec. . . . . | C. | 763.6 | 823.6 | 849.5 | 690.5 | 697.4 | 591.1 |
| | No C. | 385.0 | 651.3 | 369.0 | 495.2 | 469.3 | 486.1 |
| Standard volume in liters.. . | C. | 13.94 | 13.94 | 13.92 | 11.12 | 11.12 | 9.26 |
| | No C. | 1.394 | 1.857 | .928 | 1.159 | .926 | .926 |
| Density. . . . . | C. | .001403 | .001406 | .001409 | .001413 | .001414 | .001416 |
| | No C. | .001404 | .001408 | .001410 | .001413 | .001415 | .001416 |
| Mass evaporated per second. | C. | .02565 | .0248 | .02305 | .0231 | .02255 | .02215 |
| | No C. | .00547 | .00402 | .00355 | .00329 | .00222 | .00270 |
| Mass evaporated by C per sec. . . . . | | .02018 | .02010 | .01950 | .01989 | .01980 | .01965 |
| Heat in calories per sec. . . . . | | 1.010 | 1.010 | 1.010 | 1.010 | 1.010 | 1.010 |
| Heat of vaporization. . . . . | | 50.15 | 50.25 | 51.25 | 50.30 | 50.50 | 50.85 |

[1] Shearer, PHYSICAL REVIEW, December, 1903.

[2] When current is passing.

[3] With no current.

March 2, 1904. Bar. { 74.07 cm. / 73.80 cm. } Room. temp. { 22.90° C. / 21.40° C. }

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Voltmeter about standard coil | | .700 | .700 | .699 | .700 | .700 | .700 |
| Voltmeter about heating coil | | 6.25 | 6.25 | 6.24 | 6.25 | 6.25 | 6.25 |
| Temp. of entering gas | C.[1] | 23.02° | 24.15° | 23.23° | 22.50° | 19.10° | 19.20° |
| | No C.[2] | 23.68° | 23.75° | 23.00° | 21.50° | 19.12° | 19.25° |
| Temp. of gas leaving | C. | 23.60° | 23.80° | 22.80° | 22.08° | 18.67° | 19.00° |
| | No C. | 23.80° | 23.60° | 22.75° | 21.25° | 18.42° | 18.83° |
| Vol. in liters | C. | 20.50 | 19.50 | 16.0 | 12.0 | 12.0 | 11.50 |
| | No C. | 1.75 | 1.50 | 1.50 | 1.25 | 1.25 | 1.25 |
| Per cent. O. | | 79.0 | 85.0 | 87.8 | 89.6 | 79.6 | 84.0 |
| Time in sec. | C. | 1,038 | 1,031 | 870.1 | 651.1 | 623.7 | 619.3 |
| | No C. | 522 | 505.7 | 624.7 | 572.8 | 436.1 | 518.0 |
| Standard volume in liters | C. | 18.30 | 17.42 | 14.34 | 10.77 | 10.91 | 10.44 |
| | No C. | 1.563 | 1.341 | 1.348 | 1.127 | 1.137 | 1.136 |
| Density | C. | .001391 | .001402 | .001405 | .001411 | .001393 | .001400 |
| | No C. | .001396 | .001404 | .001409 | .001412 | .001396 | .001401 |
| Mass evaporated per sec. | C. | .02461 | .0237 | .02316 | .0230 | .0240 | .0236 |
| | No C. | .00418 | .003725 | .003045 | .00288 | .00364 | .00306 |
| Mass evaporated by C. per sec. | | .02043 | .01997 | .02011 | .02011 | .02036 | .02030 |
| Heat in calories per sec. | | 1.044 | 1.044 | 1.038 | 1.044 | 1.044 | 1.044 |
| Heat of vaporization | | 51.10 | 52.25 | 51.60 | 51.80 | 51.30 | 51.40 |

TABLE II.

| Per Cent. O. | H. of V. | Per Cent. O. | H. of V. | Per Cent. O. | H. of V. |
|---|---|---|---|---|---|
| 19.6 | 50.20 | 61.9 | 51.0 | 87.0 | 50.25 |
| 21.8 | 50.30 | 66.9 | 51.40 | 87.8 | 51.60 |
| 24.4 | 49.70 | 70.3 | 51.80 | 89.2 | 51.25 |
| 27.0 | 49.8 | 72.4 | 50.40 | 89.6 | 51.80 |
| 27.0 | 51.7 | 72.4 | 51.70 | 90.8 | 50.30 |
| 31.8 | 51.1 | 74.6 | 51.70 | 91.3 | 51.0 |
| 35.2 | 50.20 | 76.1 | 51.60 | 91.8 | 50.50 |
| 38.2 | 50.50 | 76.4 | 51.60 | 92.6 | 51.40 |
| 38.8 | 50.15 | 78.8 | 52.30 | 93.0 | 50.85 |
| 43.0 | 51.60 | 79.4 | 51.10 | 93.6 | 51.60 |
| 44.6 | 49.10 | 79.6 | 51.30 | 94.8 | 52.20 |
| 49.5 | 51.6 | 82.0 | 49.80 | 95.4 | 51.0 |
| 50.4 | 48.65 | 84.0 | 51.40 | 95.7 | 51.50 |
| 53.0 | 51.50 | 85.0 | 52.25 | 97.2 | 52.20 |
| 56.5 | 50.80 | 85.4 | 50.15 | 97.6 | 54.10 |

Total average, 50.966.

[1] When current is passing.

[2] With no current.

expect that the heats of vaporization of mixtures of the two would vary directly as the amount of each constituent in the mixture. That this is not the case is quite plainly shown in table II, which gives the heats of vaporization for the various mixtures examined, and by the curve plotted from the data (see Fig. 2).

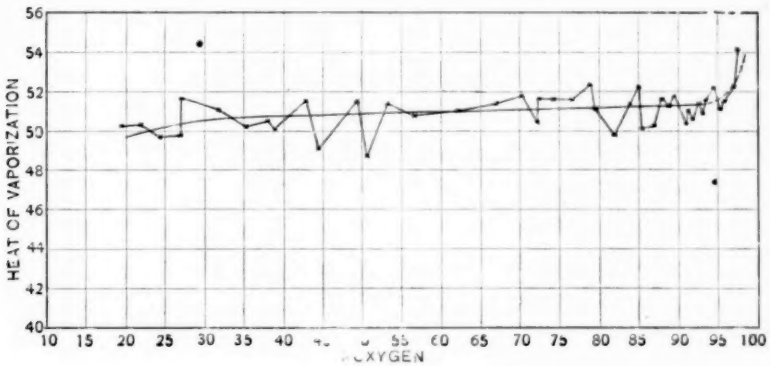


Fig. 2.

Only the two highest per cents. given show evidence of a rise in value, the remainder agreeing quite closely with the average 50.966, and it will be observed that this value compares favorably with the two values 50.7 and 50.8 as determined by Behn.[1] There seems to be a slight indication of a drop toward 49.25 in the lower per cents. and a decided indication of a rise after reaching about 97 per cent. But the change in heat of vaporization is in no way proportional to the change in composition.

The following suggestions are offered as possibilities only, it being borne in mind that we are dealing with an unstable system, and thermodynamic reasoning is difficult to apply.

We cannot regard this apparent change of heat of vaporization as a change in the property of the mixture, similar to the fluctuation of resistances or melting points of alloys, for we are dealing with energy. To convert one gram of liquid oxygen and one of liquid nitrogen into the gaseous conditions and then mix the resulting gases requires the same energy as if we first mixed the two liquids and then vaporized them, providing the initial conditions and the final conditions are the same in both cases. But since a

[1] Ann. der Physik., 1–1900, p. 270.

different quantity of heat is used to vaporize the mixture from what would be required to vaporize the components separately, it is evident that the difference between the two quantities of heat must appear in the second case other than as heat of vaporization.

A boiling mixture differs from a simple liquid in that neither of the constituents is at its own boiling point. Thus in liquid air, the nitrogen is superheated and the oxygen undercooled. Suppose we have a gram of liquid nitrogen at its boiling point, 77.5° absolute, to vaporize it at 77.5° and heat the vapor to a temperature "$t$," midway between 77.5° C. and 90.6° (boiling point of oxygen) would require

$$(49.25 + dt_n S_n) \text{ calories}$$

where $S_n$ is the specific heat of nitrogen gas at constant pressure and $dt_n$ is the change in temperature of the gas. Now first superheat the gram of nitrogen to "$t$"° C. and then vaporize, the pressure to remain constant. The heat necessary is $(dt_n S_n' + H_n)$ calories, where $S_n'$ = sp. heat of liquid nitrogen and $H_n$ the heat of vaporization at "$t$°". Since in the two cases we have initial and final conditions identical, the above quantities of heat must be equivalent or

$$49.25 + dt_n S_n = dt_n S_n' + H_n$$

or

$$H_n = 49.25 - dt_n(S_n' - S_n),$$

where $(S_n' - S_n) = \Delta_n$ is the difference between the specific heat of liquid and gaseous nitrogen.

In the same way the oxygen is undercooled and we have

$$H_0 = 60.8 + dt_0(S_0' - S_0)$$

Let $(S_0' - S_0) = \Delta_0$ as above.

If $\Delta_n$ be positive, $H_n$ is less than 49.25; and if $\Delta_0$ be positive, $H_0$ is greater than 60.8. Mixtures containing $N$ per cent. of nitrogen and $O$ per cent. of oxygen would have a heat of vaporization $H$ equal to

$$H = N(49.25 - dt_n \Delta_n) + O(60.8 + dt_0 \Delta_0)$$

or

$$H = H_c - (Ndt_n \Delta_n - Odt_0 \Delta_0)$$

where $H_c$ represents the heat of vaporization proportional to the amount of the constituents of the mixture.

From Baly's curve (see Fig. 1) $dt_nN$ is always in excess of $dt_0O$, as is shown from the following table computed from the curve. (The numbers are relative, not absolute.)

TABLE III.

| $dt_nN$ | $dt_nO$ | Per Cent. O. | $dt_nN$ | $dt_nO$ | Per Cent. O. |
|---|---|---|---|---|---|
| 246 | 153 | 15 | 401 | 231 | 55 |
| 304 | 186 | 20 | 380 | 216 | 60 |
| 352 | 210 | 25 | 350 | 201 | 65 |
| 392 | 225 | 30 | 318 | 175 | 70 |
| 409 | 238 | 35 | 278 | 150 | 75 |
| 420 | 244 | 40 | 235 | 112 | 80 |
| 423 | 243 | 45 | 192 | 85 | 85 |
| 415 | 240 | 50 | 126 | 45 | 90 |

Hence if $\Delta_n$ equals or exceeds $\Delta_0$, $H$ is less than $H_c$. This is shown by the accompanying curve, Fig. 3, plotted between per cents. of oxygen and $(dt_nN - dt_0O)$.

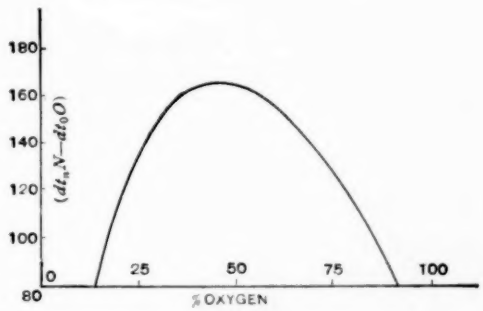

Fig. 3.

Since both $dt_nN$ and $dt_0O$ are multiplied by constants $\Delta_n$ and $\Delta_0$ respectively, the relative variation of the term $(dt_n \cdot N \cdot \Delta_n - dt_0 \cdot O\Delta_0)$ with per cents. of oxygen would be given by a similar curve. $\Delta_n$ and $\Delta_0$ not being definitely known, this curve could not be accurately drawn. However an approximation could be made and with this approximation the curve in Fig. 4 was drawn between $H$ (as computed from the above equation) and per cents. of oxygen.

This curve tends to flatten out between say 20 per cent. and 70 per cent. oxygen. By assuming proper values of $\Delta_n$ and $\Delta_0$ it could be made a straight line parallel to the $x$-axis for a considerable distance, and hence agree with the observed results.

A second point to be noted is that as the liquid boils away and becomes richer in oxygen its temperature is increased. For thus raising the temperature of the whole boiling liquid, considerable

heat is necessary. In obtaining the final results no correction was made for this quantity. The relative error is shown as follows:

During one of the "runs," the composition of the mixture changed from 43.8 per cent. to 45.4 per cent. oxygen, which from Baly's temperature curve corresponds to a change in temperature of 0.25°C. At this point the mass of the boiling liquid was about 400 grams. Its specific heat (according to Dewar) is about 0.4. That is, about 40 calories were used to heat the boiling liquid. A

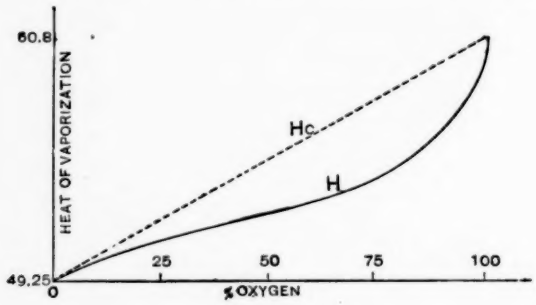


Fig. 4.

total of 1,051 calories was supplied during the run. The heat of vaporization was computed as 1051/21.3 or 49.5, whereas it should really be (1,051 − 40)/21.3 or 47.5, since 40 grams did not go toward vaporization.

Further this correction is greater with lower per cent. than with higher, for at 15 per cent. oxygen a variation of 1 per cent. changes the temperature 0.2°C., while at 90 per cent. oxygen a variation of 1 per cent. gives only 0.1° change in temperature. That is, if the heat of vaporization appears to remain constant, as found in this experiment, the real heat of vaporization, applying this correction, would increase with rising per cents. of oxygen.

In conclusion, we wish to express our obligations to Professor Shearer for his many suggestions in arranging our apparatus and for his general supervision of our work.

Physical Laboratory, Cornell University,
May, 1904.

# THE COEFFICIENT OF EXPANSION OF NICKEL AT ITS CRITICAL TEMPERATURE.

By H. M. Randall.

IRON at its critical temperature shows other remarkable physical changes, such as the phenomena of recalescence, a fall in the temperature coefficient of electrical resistance, and a marked change in its thermo-electric properties. A decided change in the nature of iron seems to occur at this temperature.

Nickel has a similar critical temperature, but its loss of magnetization at this temperature is much more gradual than in the case of iron, and the other physical changes which might be expected to accompany this loss are not so marked as in the case of iron. Hopkinson [1] was not able to find any trace of the phenomenon of recalescence as nickel changes from the non-magnetic to the magnetic state. Recently it has been shown [2] that there is a change in the thermo-electromotive force and in the temperature coefficient of electrical resistance in this neighborhood, and Holborn and Day [3] have found that there is an anomalous change in the coefficient of thermal expansion at the critical temperature.

It was thought that the method embodied in the use of the Pulfrich interferometer would be well adapted to the determination of the exact nature of any change in the temperature coefficient of expansion at this temperature. The arrangement of apparatus employed was substantially that used by the author in the investigation of the coefficient of expansion of quartz.[4] The nickel used was in the form of a cylinder of pure nickel furnished by Zeiss, Jena, 2.2 cm. in diameter and 9.690 mm. in height. Its upper surface was plane polished, and its lower surface, parallel to the upper, was ground away with the exception of three equidistant feet. This cylinder was mounted on the quartz bed-plate within the

[1] Proc. Roy. Soc., Vol. XLIV., 1888, p. 317.

[2] Campbell, Elect. Rev., 48, June 14, 1901, p. 1014–1015; also, Harrison, Nature, October 31, 1901; Phil. Mag., February, 1902.

[3] Ann. der Phys., 309, p. 104.

[4] Phys. Rev., January, 1905.

quartz ring, one of its feet having been ground away till its upper surface had the proper inclination to the lower surface of the cover plate to give interference fringes of the desired width. As the air layer had a thickness of approximately one tenth of a millimeter, the red and blue fringe systems of hydrogen vapor were very bright and they were used together with the green mercury system. At the higher temperatures the polished surface of the nickel became colored a deep blue, the result being that the red was so poorly reflected that its fringe system could no longer be used.

If $\alpha$ be the coefficient of expansion of nickel to be determined, and $\beta$ be the known coefficient of expansion of the quartz ring, while $N$ and $L$ represent the lengths of the nickel cylinder and quartz ring respectively, then $d$, the change of thickness of the air layer due to a change in temperature, $t_2 - t_1$, may be represented by the following expression in which $f$ equals the number of displaced fringes, and $\lambda$ the wave-length of the spectral color used.

$$d = f\lambda/2 = N\alpha(t_2 - t_1) - L\beta(t_2 - t_1)$$

from which

$$\alpha = \frac{f\frac{\lambda}{2}}{N(t_2 - t_1)} + \frac{L}{N}\beta$$

for the mean temperature $(t_2 + t_1)/2$.

The results obtained are given in the following tables. Table I. and Curve I., represent a series of determinations extending from room temperature to 200° C. At this point the quartz apparatus seemed to have become disarranged and the apparatus was dismounted. Some three months later the work was again undertaken, the mean temperature for the first determination being in the neighborhood of 200° C. A series of results obtained at this point with alternate rising and falling temperature showed that the coefficient of expansion of the nickel was very markedly lowered by each rise and subsequent fall of temperature. To get a series of results at all regular it became necessary to have all the temperature changes take place in the same direction. Accordingly, after allowing the apparatus to stand a week at room temperatures, a second series, Table II. and Curve II., was obtained, in which the temperature intervals were all formed by increasing the temperature.

TABLE I.

| $t_1$ | $t_2$ | $\frac{t_1+t_2}{2}$ | $\alpha \times 10^6$ | $t_1$ | $t_2$ | $\frac{t_1+t_2}{2}$ | $\alpha \times 10^6$ |
|---|---|---|---|---|---|---|---|
| 20.0 | 45.5 | 32.8 | 1326 | 76.7 | 149.9 | 113.3 | 1443 |
| 45.5 | 66.2 | 55.9 | 1356 | 103.4 | 149.9 | 126.6 | 1456 |
| 53.6 | 76.7 | 65.2 | 1356 | 150.2 | 181.9 | 166.1 | 1513 |
| 76.7 | 103.4 | 90.1 | 1399 | 181.9 | 214.2 | 198.1 | 1581 |
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TABLE II.

| $t_1$ | $t_2$ | $\frac{t_1+t_2}{2}$ | $\alpha \times 10^6$ | $t_1$ | $t_2$ | $\frac{t_1+t_2}{2}$ | $\alpha \times 10^6$ |
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| 63.6 | 105.4 | 84.5 | 1362 | 274.2 | 293.0 | 283.6 | 1637 |
| 105.4 | 137.5 | 121.4 | 1405 | 283.1 | 303.9 | 293.5 | 1638 |
| 137.5 | 157.5 | 147.5 | 1442 | 293.0 | 315.4 | 304.2 | 1652 |
| 157.5 | 178.7 | 168.1 | 1471 | 303.9 | 328.4 | 316.1 | 1649 |
| 178.7 | 193.8 | 186.3 | 1491 | 315.4 | 337.6 | 326.5 | 1637 |
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| 215.2 | 239.2 | 227.2 | 1577 | 337.6 | 357.5 | 347.6 | 1635 |
| 239.2 | 260.0 | 249.6 | 1611 | 346.3 | 368.1 | 357.2 | 1645 |
| 260.0 | 274.2 | 267.1 | 1630 | 357.5 | 378.8 | 368.1 | 1640 |
| 260.0 | 283.1 | 271.6 | 1630 | 378.8 | 404.9 | 391.8 | 1664 |

As the table and plot show, in the region included between 280° and 370°, which is generally recognized as the region of the critical

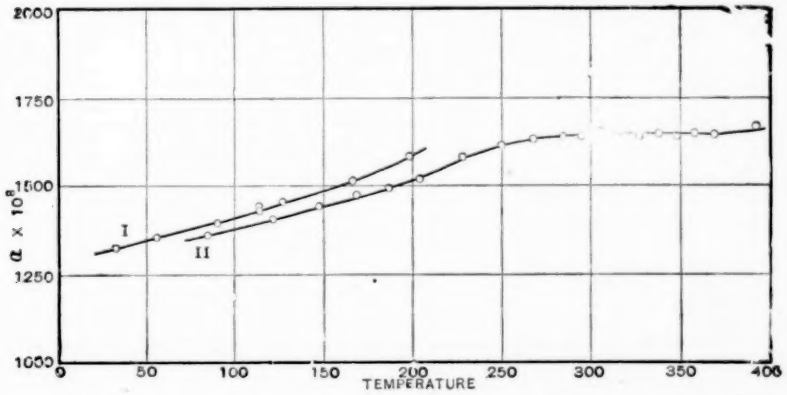


Fig. 1.

temperature, the individual values of the coefficient of expansion do not deviate from a constant value by more than would be expected from experimental error. The result obtained at 392° C. would seem to indicate that the coefficient was again increasing with a rise

quartz ring, one of its feet having been ground away till its upper surface had the proper inclination to the lower surface of the cover plate to give interference fringes of the desired width. As the air layer had a thickness of approximately one tenth of a millimeter, the red and blue fringe systems of hydrogen vapor were very bright and they were used together with the green mercury system. At the higher temperatures the polished surface of the nickel became colored a deep blue, the result being that the red was so poorly reflected that its fringe system could no longer be used.

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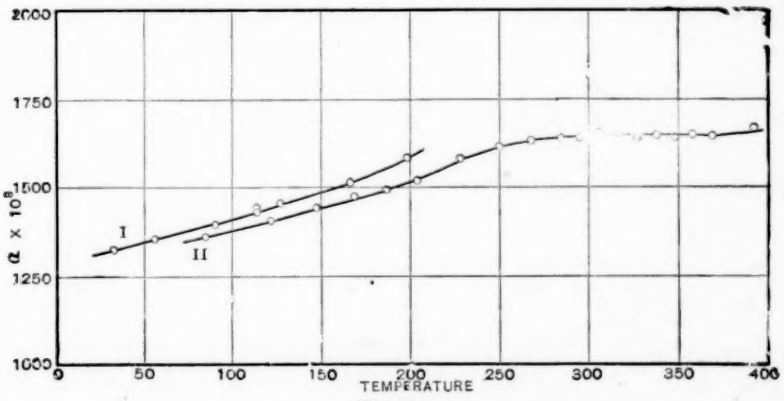


Fig. 1.

temperature, the individual values of the coefficient of expansion do not deviate from a constant value by more than would be expected from experimental error. The result obtained at 392° C. would seem to indicate that the coefficient was again increasing with a rise

in temperature. This determination cannot be given the weight of the preceding ones, however, as the nickel began to expand so unequally after passing 350° C. that the interference bands became distorted, and this distortion became so great after passing 400° C. that all further progress was stopped. Upon cooling the polished surface did not again become plane.

Harrison[1] has recently found, working with nickel wire, an anomalous expansion between 365° C. and 370° C. amounting to approximately 0.04 mm. on a length of 10 cm. An equivalent expansion of the nickel cylinder would result in a displacement of approximately 10 bands. Between 300° C. and 400° C. the number of displaced bands corresponding to a temperature interval of 20°, determined by direct counting and also by computation after the method of Abbé, did not at any time exceed three. The relative expansion is necessarily small, as the nickel cylinder and the quartz ring have nearly the same length and, in this temperature region, their coefficients of expansion do not differ greatly. An anomalous expansion represented by the passage of 10 bands could therefore hardly have escaped notice. It must be concluded then, that the nickel cylinder used in this investigation does not show, within the temperature limits employed, an anomalous expansion of the magnitude of that found by Harrison with nickel wire, and that, if such a phenomenon occurs, it must be at a temperature above 400° C.

As has been shown by Holborn and Day,[2] to get definite results with nickel it is necessary to first heat it to the maximum temperature to be employed and to allow it to cool before making measurements. The nickel cylinder, not having been thus treated before being polished, neither set of results can justly be considered as representing the coefficient of expansion of nickel, and it has not seemed worth while, therefore, to express them by means of equations. It is expected in the near future to anneal thoroughly the nickel cylinder, by repeated heatings, and repolish its surface, after which a careful determination of its coefficient of expansion will again be undertaken.

PHYSICAL LABORATORY, UNIVERSITY OF MICHIGAN,
ANN ARBOR, September 9, 1904.

[1] Phil. Mag., June, 1904.

[2] *Loc. cit.*

# THE VELOCITY OF SOUND.

By Thos. C. Hebb.

ASSUMING that the velocity of sound is an important constant, no apology is necessary for a redetermination of its value, in as much as the results obtained by different observers differ very much from one another, and in many cases the determinations of a single observer differ considerably among themselves. In the following experiment the aim has been to make the result accurate to at least one tenth of one per cent. With what success this has been attended will appear in the course of the paper.

In order to see what errors enter into the different methods of determination of this constant, the methods will be divided into (1) direct and (2) indirect. (1) can be further divided into (*a*) long distance, and (*b*) short distance.

There are several objections to the long distance method:

1. Very intense sounds — as the discharge of a cannon — are used, and it is very doubtful whether the velocity near the source is the same as that at some distance away.

2. It is practically impossible to correct for wind, temperature, and humidity with any great degree of accuracy.

3. The method of performing the experiment involves the "personal equation" of either an observer, or of a recording device as used by Regnault.

In the case of short distance methods, objections (1) and (2) are overcome, and (3) will depend on the method employed. The only method used with any success has been that proposed by Bosscha[1] and put into execution by Szathmari.[2] Although Szathmari's final result seems to be very near the true value, his individual results differ from the mean by 1 per cent.

From the great number of direct determinations of the velocity

[1] Pogg. Ann., 92, 1854, p. 485.
[2] Wied. Ann., 2, 1877, p. 418.

of sound, Wüllner[1] chooses the results of Regnault, Moll and Van Beck, Bravais and Martin, and the French Academy as the most accurate.

In Regnault's experiments a gun was used as the source of sound. In order to free the experiment from the personal equation of an observer, he had it so arranged that both the discharge of the gun at the one station, and the arrival of the sound wave at the other station, were recorded by an electrical device. Although this frees the experiment from the "personal equation" of an observer, still there is the "personal equation" of the recording device to be taken into account. In order to eliminate wind effect, simultaneous firing from both stations was used. The distance employed was between one and two miles.

In the case of the other sets of experimenters, cannons were used as the source of sound. The time between the flash of light and the arrival of the sound wave was recorded by observers. Wind effect was eliminated as in the case of Regnault. The French Academy used a distance of eleven miles, Moll and Van Beck a distance of ten miles, and Bravais and Martin nearly two miles.

In long distances like these, it is impossible to suppose that the true corrections for temperature, wind and humidity have been made. These sources of error, coupled with the errors due to personal equation and different intensities of the source are sufficient to explain the variation between the different results.

In the case of indirect observations, the wave-length is determined in a tube, making use of Kirchoff's formula connecting the wave-length in a tube with that in free air. If $\lambda$ is the wave-length in air, $L$ the wave-length in the tube, $\varepsilon$ a constant, $\gamma$ the radius of the tube, and $n$ the frequency of the source, then

$$L = \lambda \left[ 1 - \frac{\varepsilon}{2\gamma\sqrt{n\pi}} \right].$$

The objection to this method of determining $\lambda$ lies in the fact that there is good reason to believe that $\varepsilon$ is not a constant as defined by Kirchoff.[2] If the size of the tube used could be so great that the

[1] Experimental Physik, Vol. I., p. 939.

[2] J. Muller, Drude, Vol. 4, 1903, p. 331.

effect of the walls was negligible, or if the walls could be taken away entirely, the above objection would disappear and $\lambda = L$.

In the following experiment proposed by Professor Michelson, the attempt has been made to free the method from all of the above objections. The method is this :

Two paraboloids $A_{\text{I}}$ and $A_{\text{II}}$ (see cut) are placed coaxially so that waves sent out from the focus $S$ of $A_{\text{I}}$ are collected at the focus $F_{\text{II}}$ of $A_{\text{II}}$. In the one focus, $S$, is placed a source of sound and near it a telephone transmitter, $T_{\text{I}}$, while in the other focus is placed another transmitter, $T_{\text{II}}$. Each transmitter is connected in series with a battery ($B_{\text{I}}$ and $B_{\text{II}}$ in figure), and one primary of an induction coil, $I$, made with two primaries. A telephone receiver, $R$, is in series with the secondary of the coil.

Now, suppose waves are given out by the source $S$. Some pass directly to the transmitter, $T_{\text{I}}$, placed near it and set it in vibration with a definite phase relation depending on its distance from the source $S$; others being reflected to the other paraboloid, $A_{\text{II}}$, are collected at its focus, where they act on the second transmitter, $T_{\text{II}}$, and set it in vibration with a definite phase relation depending on the distance between the two paraboloids. The vector sum of these two *effects* is given in the receiver. Assuming that the intensities of these effects are the same, it is possible, by moving one paraboloid parallel to itself, to change the relative phase of the two effects so that they will alternately annul and reinforce one another. This affords a method of getting the wave-length, for having determined the position of the paraboloid for two different minima, and knowing the number of waves, and the distance between these two positions, the value of the wave-length follows immediately. If now the paraboloid can be moved one hundred waves, and if the minima can be located to one tenth of a wave, the value of the wave-length so obtained should be accurate to one part in one thousand. This assumes that the difference of phase is only a function of the distance between the paraboloids. In order to realize this, the waves reflected from the paraboloids must be plane.

## Apparatus.

The paraboloids used were 5 feet in aperture and 15 inches in focal length. They were made of plaster of paris and were accurate

of at least one tenth of an inch. They were mounted on strong trames with their axes horizontal and were provided with wheels for locomotion.

The transmitters used were disk-shaped, 3 inches in diameter and 1.3 inch thick. The aperture through which the sound entered was .3 inch in diameter.

The source of sound consisted of a tube .75 inch in diameter, closed at one end and having a stream of air blowing across the other. It was so arranged that as few overtones as possible were present. The air pressure was supplied by the pressure tubes of the laboratory. This pressure is generated at the University power-house, 600 feet away, and hence has a very constant value in the laboratory pipes. In order to further increase its constancy, the air was passed through two large gas tanks and then through cotton wool and lime. This also removed the dust and moisture. In this manner it was possible to keep the pitch of the whistle constant to one part in five thousand for a time sufficient to make an observation.

One paraboloid was movable on a track. This track was built in sections and was taken up after each experiment. The rails were 2.5 feet apart and prevented any side motion of the paraboloid.

The induction coil used with the transmitters had two primaries wound with No. 22 Cu wire, and having a resistance of .3 ohms. The secondary was also of No. 22 Cu wire and had five times the number of turns of either primary.

The experiment was performed in one of the halls of Ryerson Physical Laboratory. The hall is 120 feet long, 10 feet wide, and 14 feet high. It should be remarked that the experiment need not be restricted to this distance. In this place no wind was encountered, and the temperature remained quite constant. Six thermometers corrected for zero point were placed at equal intervals on alternate sides of the hall and the mean of their readings taken as the temperature at which the experiment was performed.

Arrangement of the apparatus: The two paraboloids $A_{\text{I}}$ and $A_{\text{II}}$ were arranged coaxially (see cut), one stationary at the end of the hall, and the other movable on a track extending from one end of the hall to the other. The stationary paraboloid $A_{\text{I}}$ had the

whistle at its focus and near it a transmitter $T_1$. Wires led from this transmitter through a battery $B_1$, to the one primary of the induction coil $I$ placed at the back of the paraboloid $A_{11}$. In the focus of $A_{11}$ was the other transmitter $T_{11}$ whose wires led through a battery $B_{11}$, to the other primary of the induction coil. Having a receiver $R$ in series with the secondary of the coil, it was possible to listen in the telephone, and move the paraboloid $A_{11}$ at the same time. In each of the primary circuits was placed a slide wire

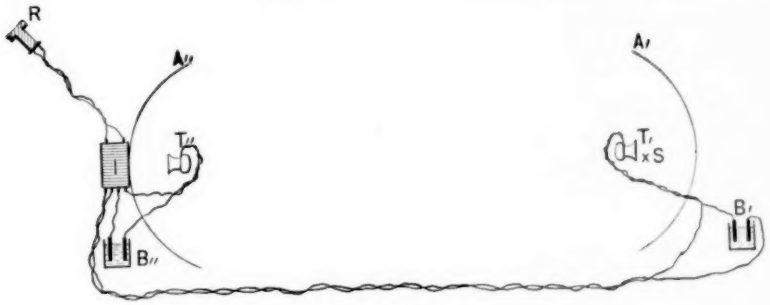


Fig. 1.

resistance, so that the intensities of the effects in the receiver could be kept equal and constant.

To perform an experiment the whistle was brought into unison with a tuning fork and the movable paraboloid having been brought to within fifteen feet of the fixed one, a position was found at which the sound in the telephone receiver was a minimum. Trouble was encountered here for the pitch of the whistle depends on the position of the movable paraboloid. This is due to the reflected wave changing the pressure at the mouth of the whistle and is overcome by changing the pressure in the air supply. This reflected wave would also have the effect of changing the phase of vibration of the transmitter near the whistle. This was reduced to a minimum by making the reflected wave as weak as possible. Other causes of change of phase in the transmitter of the movable paraboloid are the direct action of the whistle, and the action of the waves reflected from the sides of the hall. These however were very small as could be tested by putting the whistle before an open door at the end of the hall. The position of the paraboloid for which the telephone

gave minimum intensity could be determined to .5 inches. Two of these positions were found and hence the wave-length of the fork could be found.

Experimenting with a 2560 V. S. fork, the following values of the wave-length for one night were obtained:

TABLE I.

*March* 24.

| No. of Waves Beginning at 15 Feet from Source. | Distance Between Minima in Inches. | Temp. C. °. | $\lambda_t$ | $\lambda_0$ |
|---|---|---|---|---|
| 10 | 106.62 | 22.4 | 10.662 | 10.249 |
| 20 | 213.05 | 22.5 | 10.652 | 10.237 |
| 30 | 319.23 | 22.6 | 10.641 | 10.225 |
| 41 | 436.78 | 22.5 | 10.653 | 10.238 |
| 51 | 541.58 | 22.4 | 10.619 | 10.207 |
| 61 | 647.70 | 22.7 | 10.618 | 10.202 |
| 71 | 754.19 | 22.7 | 10.622 | 10.205 |
| 81 | 861.12 | 22.9 | 10.631 | 10.210 |
| 119 | 1,263.57 | 22.9 | 10.618 | 10.199 |

In these results no correction has been made for humidity, but this would not change for one night. As can be seen from the table, the values of the wave-length decrease as we proceed from the source up to a certain point, and then remain constant. This decrease cannot be explained by the experimental error, for similar results were obtained at different times.

The following table will show that even under different circumstances the value of the wave-length in the case where the whole length of the hall is used is a constant:

TABLE II.

| Date. | No. of Waves. | Distance Between Minima in Inches. | Temp. C.°. | $\lambda_t$ | $\lambda_0$ | $\delta$ |
|---|---|---|---|---|---|---|
| Mar. 22 | 80 | 848.81 | 22.2 | 10.610 | 10.203 | +.008 |
| 24 | 119 | 1,263.57 | 22.9 | 10.618 | 10.199 | +.004 |
| 25 | 117 | 1,240.82 | 22.0 | 10.604 | 10.200 | +.005 |
| 27 | 119 | 1,255.25 | 20.0 | 10.548 | 10.181 | −.014 |
| Apr. 3 | 119 | 1,255.87 | 19.7 | 10.554 | 10.192 | −.003 |
| | | | | | Mean 10.195 | |

The probable error of the mean is .003. There is however in these results a constant error due to the fact that the humidity has

not been taken into account. When these observations were taken however, the weather was still cold so that the dew point was probably below zero; and hence the above values are possibly .1 per cent. too great. It will be of interest as a check to see how the velocity of sound derived from this wave-length with the aid of the marked vibration number of the tuning fork agrees with the accepted value. The tuning fork had been kindly loaned to me by Professor Angell, Head of the Psychology Dept., and he assured me when I got it that it was not in error more than one-half a beat. Accepting these two values then we get for the velocity of sound at zero degrees:

$$U = \lambda_0 n = \frac{10.195 \times 1280}{39.37} \text{ meters} = 331.46 \text{ meters.}$$

This is about .1 per cent. less than the accepted value. It would appear from this that the decrease in wave-length as shown in Table I. is not very serious. Believing that the decrease was due to the fact that the wave-length was too long to be properly reflected, a wave six inches in length was tried. The following table shows that the trouble has been overcome:

TABLE III.

| Date. | No. of Waves Beginning 15 Feet from Origin. | Temp. C.°. | Distance Between Minima in Inches. | $\lambda_t$ | $\lambda_0$ |
|---|---|---|---|---|---|
| April 17 | 25 | 20.5 | 142.61 | 5.7044 | 5.5013 |
| " | 75 | 20.9 | 428.20 | 5.7093 | 5.5022 |
| " | 100 | 21.1 | 570.50 | 5.7050 | 5.4962 |
| 19 | 25 | 19.3 | 142.36 | 5.6944 | 5.5029 |
| " | 50 | 19.4 | 284.35 | 5.6870 | 5.4947 |
| " | 75 | 19.8 | 427.21 | 5.6961 | 5.4996 |
| " | 100 | 19.7 | 568.94 | 5.6894 | 5.4942 |
| " | 126 | 19.5 | 717.05 | 5.6909 | 5.4976 |
| " | 151 | 19.4 | 859.16 | 5.6898 | 5.4975 |
| " | 176 | 19.1 | 1,001.27 | 5.6890 | 5.4995 |
| " | 220 | 19.2 | 1,251.87 | 5.6903 | 5.4999 |

It is seen from this table that there is no tendency for the wave-length to decrease as was the case with the longer wave-length. What variations there are, are within the experimental error. This seemed to show that all constant errors were eliminated and hence

the observations given in the following table of observations were made. Unfortunately there are not more data to offer due (1) to the fact that the experiment is very trying on the nerves and (2) to lack of time. These observations were made however under different atmospheric conditions, and as the apparatus was taken down after each experiment there is little likelihood of there being a constant error of method. It might also be added that no observations have been rejected.

TABLE IV.

| Date. | No of Waves Used. | Distance Between Minima in Inches. | Temp. C.°. | Dew Point | Press. mm. | $\lambda_t$ | $\lambda_0$ | $\delta$ |
|---|---|---|---|---|---|---|---|---|
| Apr. 23 | 217 | 1,244.00 | 22.5 | 14° | 746.5 | 5.7328 | 5.4940 | +.0046 |
| " | 216 | 1,237.87 | 22.5 | 14 | 746.5 | 5.7309 | 5.4921 | +.0027 |
| 24 | 217 | 1,238.12 | 21.3 | 4 | 746.5 | 5.7056 | 5.4866 | −.0028 |
| 26 | 218 | 1,240.00 | 18.8 | 1 | 752.0 | 5.6888 | 5.4956 | +.0062 |
| 30 | 219 | 1,241.37 | 17.5 | 4 | 747.5 | 5.6684 | 5.4855 | −.0039 |
| May 1 | 216 | 1,224.37 | 17.4 | 5 | 750.0 | 5.6684 | 5.4865 | −.0029 |
| 2 | 218 | 1,235.50 | 17.4 | 4 | 750.8 | 5.6674 | 5.4855 | −.0039 |
| | | | | | | | Mean 5.4894 | |

The probable error of the mean value is .0010. $\lambda_0$ has been calculated by aid of the following formula:

$$\lambda_0 = \lambda_t \sqrt{\frac{\rho}{\rho_0}}$$

where $\lambda_0$ = wave-length at zero for dry air, $\lambda_t$ is wave-length at time of experiment, $\rho$ is the density of the air at the time of the experiment, and $\rho_0$ is the density of dry air at 0° and pressure equal to that at time of experiment.

### DETERMINATION OF THE PITCH OF THE TUNING FORK.

This consisted of three parts (1) the comparison of the high fork with one marked 512 V.D., (2) the comparison of this latter fork with a pendulum, and (3) the comparison of the pendulum with the clock. The comparisons in (1) and (2) were made by means of traces on a smoked glass disk.

1. In this case it was found that the vibrations of the high pitch fork were of too short a duration to admit of an accurate comparison by means of a trace from it. It was therefore found necessary

to use the trace given by a stile fastened to the telephone receiver diaphragm, when the latter was actuated by the whistle in unison with the fork. In order to make the trace of sufficient amplitude to be distinctly seen with a microscope, it was necessary to bring the two paraboloids close together so that the intensity of the sound at the mouth of the transmitter used was rather great. This together with the use of a large current through the transmitter gave sufficient amplitude of the receiver diaphragm to make its trace seen with a low power microscope. As stated above, the unison of the whistle and tuning fork could be obtained to one part in five thousand. The stiles whose traces were being compared were arranged close together so that the traces were in the field of view of a microscope, and so that a variation of the speed of rotation of the disk did not affect the results. The unison of the telephone and tuning fork was tested before and after an experiment.

The following three determinations were made :

TABLE V.

| Date. | Temp. C°. | $n_1$ | $n_2$ | $n_1/n_2$ | $\delta$ |
|---|---|---|---|---|---|
| May 9 | 19.5 | 1,041 | 224 | 4.6473 | .0007 |
| 12 | 19.3 | 1,008 | 217 | 4.6452 | .0014 |
| 13 | 19.0 | 781 | 168.05 | 4.6474 | .0008 |
| | | | Mean | 4.6466 | .00097 |

$n_1$ is the number of waves of the telephone diaphragm which corresponded to $n_2$ of the 512 V. D. fork. The above numbers were obtained by counting the number of waves between two points at which the crests of the two waves coincided. These positions could be determined to .03 of a 512 wave.

2. The pendulum used in this case was made to close a current which acted through an electromagnet bearing a stile. A complete period of the pendulum was about 2 sec. Using the usual precautions with the mercury contact, etc., the following two values for the number of waves of the tuning fork between the two breaks of a complete oscillation of the pendulum were obtained.

| May 17, | Temp. 18.3. |
|---|---|
| First observation gave | 1,017.7 |
| Second observation gave | 1,017.7 |
| Mean | 1,017.7 |

The position of the break of the electromagnet could be located to .1 of a wave.

3. The pendulum was compared with the Howard astronomical clock of the laboratory. In two separate comparisons when the number of swings of the pendulum in one hour was obtained, the following values for a half swing were found :

$T = .99498$

$T = .99485$

Mean $T = .99492$.

Therefore the pitch of the high tuning fork at 18.3 is

$$N_{18.3} = \frac{4.6466 \times 1017.7}{2 \times .99492} = 2{,}376.5.$$

Using McLeod and Clarke's formula for the change in pitch of a tuning fork due to change in temperature, viz. :

$$n_t = n_0 (-.00011t)$$

the following table which is derived from Table IV. is obtained :

TABLE VI.

| Date. | Temp. C.°. | $\lambda_0$ | $n_t$ | $v_0 = n_t\lambda_0$ | $\delta$ |
|---|---|---|---|---|---|
| April 23 | 22.5 | 5.4940 | 2,375.4 | 331.48 | +.19 |
| 23 | 22.5 | 5.4921 | 2,375.4 | 331.37 | +.08 |
| 24 | 21.3 | 5.4866 | 2,375.8 | 330.95 | −.34 |
| 26 | 18.8 | 5.4956 | 2,376.4 | 331.72 | +.43 |
| 30 | 17.5 | 5.4855 | 2,376.7 | 331.15 | −.14 |
| May 1 | 17.4 | 5.4865 | 2,376.7 | 331.21 | −.09 |
| 2 | 17.4 | 5.4855 | 2,376.7 | 331.15 | −.14 |
| | | | | Mean 331.29 | .20 |

The probable error of the mean is .04.

This value lies a little below the theoretical value obtained by assuming the ratio of the specific heats to be that obtained by Röntgen, viz., 1.405. Using this value of $\gamma$, 331.8 is obtained for the velocity of sound. It also lies below the mean of the four best determinations which according to Wüllner[1] are :

[1] Experimental Physik, Vol. I., p. 939.

| | |
|---|---|
| Regnault, | 330.7 |
| Moll and Van Beck, etc., | 332.7 |
| Bravais and Martin, | 332.4 |
| French Academy, | 331.2 |
| Mean, | 331.75 |

In conclusion I wish to thank Professor Michelsen for proposing the problem, and for his valuable advice at all times freely given.

I also wish to thank Professor Millikan for his kind assistance.

RYERSON PHYSICAL LABORATORY.

# THE ARC IN A MAGNETIC FIELD.

BY. C. D. CHILD.

THE following is an account of certain phenomena observed when the electric arc is placed in a magnetic field. A preliminary description of these were given before the American Physical Society in March, 1904, and an abstract of the work appeared in the PHYSICAL REVIEW.[1]

*The First Experiment.*— The connections of the first experiment are shown in Fig. 1. *A* and *B* are two carbons 12 mm. in diameter. *c* and *d* are carbon pencils 3.1 mm. in diameter placed near to the side of the arc. These are connected through a millivoltmeter having a resistance of 177 ohms. This voltmeter was quite dead beat and the period was about 1 sec. An electromagnet is placed near the arc and produces a field normal to the plane of the figure.

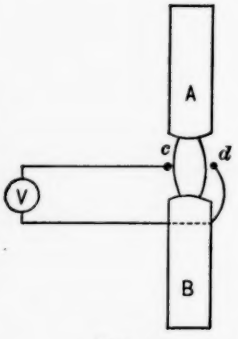

Fig. 1.

It was found quite impossible to place *c* and *d*, so that the potential difference between them continued to be zero, or even to place them so that it was constant. The pointer of the voltmeter would at times remain quite for several seconds and then change suddenly, sometimes changing only a slight amount and at other times reversing. It is impossible, at least, in air to keep the potential difference constant, because *c* and *d* are consumed rapidly and before the arc reaches a quiescent condition they are burnt away from it. But even though the reading of the voltmeter was not constant, it was evident that whenever there was a sudden change in the magnetic field, there was a corresponding change in the reading of the voltmeter. If the direction of the field was such that the arc was

[1] PHYS. REV., 18, 370.

displaced to the right, $d$ would become positive, as compared with its former condition, and $c$ negative; if to the left, $c$ would be positive, and $d$ negative. If in the first example $d$ was already positive, it would become more so, and if negative, it would become less negative.

*Electrometer Substituted for Voltmeter.*— It was soon found that a voltmeter was not an ideal instrument for the purpose. As has been stated its resistance was only 177 ohms and the apparent resistance between the two pencils in the arc, even when they were very hot, appeared to vary from a few ohms to several hundred. On the other hand the period of such electrometers as were at hand was so great that the condition of the arc changed much during a reading. As has been stated, the rapid burning of the electrodes and especially the exploring pencils made it desirable to take readings as rapidly as possible. Moreover a voltmeter is a much more convenient instrument to use than an electrometer. As a result the experiments were nearly all performed first with a voltmeter and more careful measurements were afterwards taken with an electrometer. The electrometer used had a period of 16 seconds and was very nearly dead beat.

Thus the experiment already described was repeated using an electrometer instead of a voltmeter, and the same results were observed as before.

*Modifications of this Experiment.* — As will be seen later there is doubt about the real significance of this phenomenon, and consequently many variations of the experiment were tried. One of these may be described as follows: It has been shown that $d$ in Fig. 1 has a higher potential when the arc is deflected by a magnet to the right and a lower one when it is deflected to the left. If $A$ is the positive carbon, then the potential difference between $A$ and $d$ should decrease when the arc is shifted to the right, and increase when shifted to the left. Such changes were, indeed, found to occur.

On the contrary when the voltmeter was connected between $B$ and $d$, the readings increased on deflecting the arc to the right, and decreased on deflecting it to the left. The opposite results followed when the exploring pencil was placed at $c$ instead of at $d$.

A slight modification of this experiment was made by connect-

ing a shunt between $A$ and $B$ and connecting $d$ to some point of this shunt. The observation again indicated that the potential of $d$ is increased when the arc is deflected to the right, and diminished when it is deflected to the left.

The electric force through the arc was also measured both with and without the magnetic field, and no difference could be detected in the two cases.

*Rapid Reversing.* — Since the potential difference between $c$ and $d$ was not for any length of time constant, an arrangement was used by which the ordinary fluctuations of this potential were eliminated and those produced by the magnetic field were retained. To do this the connections between $c$ and $d$ and the voltmeter were rapidly reversed. The voltmeter would thus normally show no deflection. But at the same time that these connections were reversed those running to the electromagnet were also reversed. The effect due to the magnetic field was thus shown as before.

The reversals were made by a wheel, as shown in Fig. 2. This was made of fiber, and was 10 cm. in diameter. It is a combination of a commutator and a collecting ring. 1 and 2 are connected to the carbon pencils, 3 and 4 to the voltmeter. With each revolution of the wheel two reversals would occur.

Fig. 2.

A similar arrangement was used for reversing the magnetic field the reversal of one circuit taking place at the same time as that of the other.

With this arrangement a reading of the voltmeter was attained which was at least always in the same direction, and the reading was reversed when the direction of the current through the arc was reversed, or the connections to the reversing wheel for the electro-magnet. The readings indicated as before that the current passed from $d$ through the voltmeter to $c$, when the arc was deflected to the right.

The arc was then removed and $c$ and $d$ were connected by copper wires. No effect, whatever was then observed. This shows that the effect was not due to any E.M.F. induced by the alternations of the magnetic field.

*Arc Deflected Mechanically.* — But though the accuracy of the first observations was thus proven, there was still the possibility that

the magnetic field was not the direct cause of the observed effect, that rather the mechanical motion of the arc was the direct cause. To test this the arc was blown to one side by a blast of air. This usually produced a smaller reading of the voltmeter whatever that reading might be, when that instrument was used. This was due to the fact that when the arc is blown to one side, the resistance between the pencils is increased.

When an electrometer was used, it was found that with the arc blown sufficiently to one side, so that one of the pencils became much cooler than the other, the cool one always became charged positively. Further investigation showed that the space about the arc had a higher potential than the arc itself and that when the arc was blown away from one of the pencils, that pencil became charged positively, merely because it was assuming the potential of the surrounding space.

The difference between the phenomenon in this case and that when the arc was deflected by a magnet was especially noticeable when the arc was entirely displaced from one of the pencils. Such a pencil was charged negatively when the arc was deflected by a magnet, and positively when deflected by a blast of air.

Moreover as long as both carbons were kept very hot no regular effect could be detected by blowing the arc, while deflecting it by a magnet always produced an effect.

The difference in these results was also strikingly shown when the potential difference between one of the pencils and one of the electrodes was examined. It has already been stated that in certain cases such potential differences increased, while in others they decreased, when the arc was deflected by a magnet. When it was blown to one side the potential difference in every case increased, provided the exploring carbon was kept in that part of the arc where the conductivity was good.

*Further Test of Effect Produced by Mechanical Motion.* — In order to test further the suggestion that the effect here studied was produced by the mechanical motion of the arc and not by the magnetic field, a fan was arranged which blew the arc first in one direction and then in the other. This arrangement is shown in Fig. 3. *W* is a wheel taking the place of the reversing wheel previously de-

scribed. $P$ is a pin fastened to the wheel and moving in the slit $RH$. $H$ is a hinge. At $R$ two fans $F$ and $F'$ are fastened at $R$ and the arc is placed between them.

The reversals of the motion of the fan occurred at the same time as those of the connections to the voltmeter, so that this arrangement produces the same effect as far as mechanical motion is concerned as the magnetic field. But no deflection of the voltmeter was observed even when the velocity of the fan was increased until the arc was blown out.

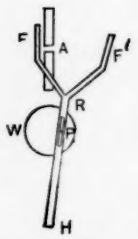

Fig. 3.

*Varying the Rapidity of the Reversals.* — The first thing varied was the period of the reversals. This tests again the possibility that the effect is due to the unequal heating of the two carbon pencils. Though the temperature of the carbon of the arc follows very closely the fluctuations of the arc, there was the possibility that the reversals could be produced so rapidly that the effect would decrease. It was found, however, that increasing the rapidity of the reversals did not diminish the effect. Thus when the number of reversals was increased from 4 per second to 25 per second, the reading of the voltmeter diminished from .35 volts to .30 volts. Several observations were made, but this is the average, and further investigation showed that this decrease in the reading is not due to any change in the conditions of the arc, but to the self-induction of the electro-magnet and the voltmeter and to changing contact resistance.

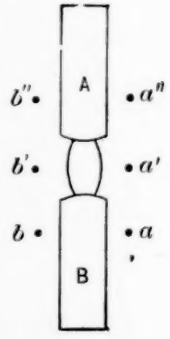

Fig. 4.

*Distance Between Exploring Pencils Varied.* — The distance between the exploring carbons was then varied. It was found, however, that the results obtained when the pencils were more than 7 or 8 mm. apart depended very largely on the height of the exploring carbons. Thus at $ab$ the potential difference was small, at $a'b'$, it was much larger, while at $a''b''$, it was reversed. It was probably small at $ab$ because there they were only barely touched by the arc. At $a'b'$ they were more directly in the arc. At $a''b''$ the effect appeared to be due to the mechanical motion of the arc, and

not to any magnetic cause. If the arc is blown to the right $b''$ will be in the hot gases arising from the arc more than $a''$. It will consequently have more nearly the potential of the arc, and as we have seen the arc is negative as compared with the surrounding space.

*Current through Arc Varied.* — The current through the arc was varied from 5 to 25 amperes, but no definite change in the potential difference between the exploring pencils was discovered. According to the measurements taken the change was at least less than one part in twenty.

*Length of Arc Varied.* — The effect was found to increase when the length of arc was increased. The following observations were taken, using an electrometer, the alternating apparatus, a current of 12 amperes, and a distance between the pencils of 9 mm.

TABLE I.

| Length of Arc. | Potential Differences Between Pencils. |
|---|---|
| 3 | 1.7 |
| 6 | 2.5 |
| 9 | 3 |

*Strength of Field Varied.* — Next the strength of the field at the arc was varied. To determine the strength of the magnetic field a long solenoid 3.5 sq. cm. in cross section was wound with 13.8 turns per cm. One hundred turns of wire were wound about this and the throw of a ballistic galvanometer was observed when a given current was sent through the primary turns. This was compared with the throw produced by 100 turns at a given distance from the electromagnet which had been used with the arc. The effects observed in this way with the electromagnet at different distances from the arc are given in Table II.

TABLE II.

| Distance from Solenoid. | Magnetic Field. | Potential Difference Between Pencils. |
|---|---|---|
| 6 | 13.4 | 4 |
| 9 | 7.5 | 3 |
| 12 | 5 | 2.2 |
| 15 | 3 | .8 |

The distances between the arc and the end of the magnet are given in column 1, the strength of the field in electromagnetic units in column 2, and the potential difference between the exploring car-

bons in volts in column 3. The current through the arc was 12 amperes and the length of the arc was 5 mm. These readings were taken with the electrometer and alternating apparatus.

*Phenomenon Dependent on Quality of Carbons Used.* — While making these observations it was found that the amount of the effect depended very largely on the kind of carbon used. Those first used were solid, but in changing the apparatus cored carbons were unintentionally substituted, and for some time none of the experiments which had been performed could be repeated. In time it was found that it was the change in carbons which caused the difference. It was found that, in general, solid carbons gave a greater effect than cored. It happened that the first ones used gave a greater effect than any others investigated. I do not know their make, as they had been in the laboratory a number of years.

*Different Substances in the Arc.* — Further investigations showed that the effect was diminished very much when certain salts were placed in the arc. Indeed with some it vanished entirely. In testing this a hole was drilled for 2 or 3 cm. into the carbon and this was filled with the salt to be investigated. Only a very small effect was found with NaCl, and none whatever with $KNO_3$. Moreover, no effect was found with the arc between iron or copper terminals.

An attempt was made to use graphite terminals, but it was found difficult to maintain any arc with the terminals far enough apart to insert the exploring carbons, even when using a voltage of 200 volts, and when the exploring carbons were inserted a very small magnetic field was sufficient to extinguish the arc. As a result no measurements were made with graphite terminals.

*Phenomena in a Vacuum.*— It was also found that the effect disappeared in a vacuum. The observations were made in a tube which has been described in a previous article [1] the exploring pencils were 1.6 mm. in diameter. They were led into the tube by the method which had been previously used in examining the electric force in the arc. A series of readings thus taken are given in Table III., the distance between the exploring carbons was 5 mm. and they both became white hot while in the arc. The arc was 7 mm. long. The current was 10 amperes. The end of the electro-

[1] PHYS. REV., 19, 117.

magnet was 7 cm. from the arc, making the field at the arc 10 electomagnetic units. Column 1 gives the pressure of gas in millimeters of mercury. Column 2 gives the difference in potential between the exploring carbons in volts.

TABLE III.

| Pressure of Gas. | Potential Difference Between Pencils. |
|---|---|
| 750 | 3.3 |
| 550 | 2.5 |
| 400 | 1.9 |
| 300 | 2 |
| 200 | 1.2 |
| 100 | .7 |
| 50 | .4 |
| 20 | .2 |

At 6 mm. pressure no effect could be detected either with an electrometer or with a voltmeter. A galvanometer having a constant of $3.6 \times 10^{-5}$ ampere per scale division was also used and it was even then impossible to detect any effect. The galvanometer had a resistance of 445 ohms and a period of 13.5 sec.

It seemed useless to try a more sensitive galvanometer. For without any magnetic field the potential differences betwen the two pencils in the arc fluctuated from zero to .2 volt and one cannot well eliminate an effect of this size so completely as to study one which was less than .001 volt.

*Possible Explanations.*— During the experiments it was noticed that a magnetic field has less effect in deflecting the arc to one side when it is in a vacuum than when in air. Thus the field used with the vacuum was 15 units and it scarcely affected the appearance of the arc, while it extinguished it entirely when using the same length of arc and same current in air. It was thought at first that this might possibly be the cause of the disappearance of the effect in a vacuum. But the effect also decreases when NaCl was placed in the arc, and in that case the arc is blown fully as much to one side as with ordinary conditions.

In a recent article by Weintraube[1] it is stated that when a magnetic field is produced about a mercury arc in a vacuum the bright spot at the cathode moved in one direction, while the vapor of the

[1] Phil. Mag. (6), 7, 95.

arc moved in the opposite. Special notice was, therefore, taken of the arc in air and it was found that generally the bright spot did not move at all, when the magnetic field was produced, but that when it did, it moved in the same direction as that of the vapor of the arc.

It is evident that an explanation for these phenomena must explain the many cases where the effect disappears. We find that wherever it disappears the drop in potential at the anode is much smaller than it is with carbons where it is large. Thus with a solid carbon the drop in potential between the anode and an exploring pencil placed as near to it as possible was 41 volts. With a cored carbon with which the effect was scarcely noticeable the drop was 30 volts. With the iron with which experiments were performed it was 16 and with copper 21 volts. In a previous article[1] it has been shown that with the carbons there used the drop was 50 volts with a pressure of 730 mm. of mercury, 43.5 at one of 200 mm., and 26.8 at 10 mm. Thus is all cases where the effect disappears the drop in potential at the anode is much diminished.

There is another set of facts with which the phenomena may be connected. It was shown in a previous article that with ordinary carbons the positive discharge from an arc to a surrounding cylinder was greater than the negative.[2] This indicated that the positive ions had the greater velocity. When salts were placed in the arc, or when other electrodes than carbon were used, the negative discharge was approximately as great, or even greater than the positive. In a more recent article[3] it was shown that in a vacuum the negative discharge again became the greater. So that in all cases which have thus far been studied when the drop in potential at the anode is small, the discharge from an arc to a cylinder becomes relatively larger and the effect due a magnetic field disappears.

In view of the complications existing in the arc it may be unwise to offer any explanation of the phenomena here described, and yet an attempt to do so may lead at least to further investigation, and I would, therefore, suggest the following:

[1] Phys. Rev., 19, 120.
[2] Phys. Rev., 12, 137.
[3] Phys. Rev., 19, 125.

The usual equation for the Hall effect is $Z = \frac{1}{2} XH(u - v)$ where $Z$ equals the electric force due to the Hall effect; $X$, the impressed electric force; $H$, the magnetic field; and $u$ and $v$, the velocities of the positive and negative ions respectively for unit electric force. The directions of the quantities are those indicated in Fig. 5. If we should apply these equations to the arc we would conclude that the positive ions have velocities enormously greater than the negative.

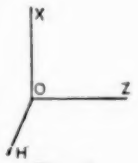

Fig. 5.

However, in deriving this equation the assumption is made that the ions have no component of motion perpendicular to the $X$ direction, whereas in the arc there is such motion. It is the motion of the ions to one side which causes the arc to be deflected. This equation, therefore, can not be said to hold in this case. However, from physical considerations we shall be led to the same general conclusion as when treating the matter mathematically, that is, we are led to believe that the positive ions have the greater velocity.

In the arc both positive and negative ions are deflected, but those having the greater velocity will be deflected most. This can be shown as follows:

Let $e$ = the charge on an ion.

Let $m$ = the mass of the ion.

Let $H$ = the magnetic force in the direction of the $Y$ axis.

Let $X$ = the electric force in the direction of the $X$ axis.

Let $x$ and $z$ be the distances from the origin along the $X$ and $Z$ axes to the point where the ion is at a given instant.

If the $X$, $Y$, and $Z$ axes are at right angles to each other and if the ion starts from a condition of rest at the origin

$$y = \frac{X}{\omega H}(\omega t - \sin \omega t)$$

and

$$x = \frac{X}{\omega H}(1 - \cos \omega t)$$

where

$$\omega = \frac{He}{m}.$$ [1]

[1] Thomson's Conduction of Electricity through Gases, 88.

If $t_1$ is the time necessary for the ion to move over its mean free path and $x_1$ and $y_1$ the corresponding values of $x$ and $y$,

$$\frac{y_1}{x_1} = \frac{\omega t_1}{3} - \frac{(\omega t_1)^3}{90} + \cdots.$$

But the average velocity during this time, if there had been no magnetic force would have been $Xet_1/2m$. This will be the same as the average velocity for any number of mean free paths and if we let $v_a$ equal the average velocity when the electric force is one

$$v_a = \frac{et_1}{2m}.$$

Then

$$\omega t_1 = \frac{Het_1}{m} = 2Hv_a$$

and

$$\frac{y_1}{x_1} = \frac{2Hv_a}{3} - \frac{(2Hv_a)^3}{90} + \cdots.$$

If now $v_a = 1000$ cm. per sec. for one volt per sec. and $H = 10$ E.M. units the ratio between the first and second term equals $\frac{2}{3}\,10^{-9}$, so that for such cases as are here considering $y_1/x_1 = \frac{2}{3}Hv_a$.

There will be $1/x_1$ of the collisions per cm. so that the total deflection in going 1 cm. in the $x$-direction $= y_1/x_1 = \frac{2}{3}Hv_a$ which is greater, the greater the average velocity of the ion.

This case is evidently quite different from that of cathode rays. There practically all of the deflection takes place while the ion is moving with uniform velocity, while here it is supposed to occur where the velocity is uniformly accelerated.

If then the negative ions are deflected most, the part of the arc to the right, namely, in the $Z$ direction, will have an excess of negative ions, and the potential at the right will be lower than that at the left. If on the other hand the positive ions have the greater velocity, then the potential at the right will be positive as compared with that on the left. This latter is what was found to be the case. The potential on the side toward which the arc was deflected was greater than that from which it was deflected. We are, therefore, led to believe that in the arc the positive ions move more rapidly.

That this should be so in a place as hot as the electric arc is not what one would have expected. Yet reasons for believing that this is possible have been given elsewhere.[1] When the anode is very hot we may expect ebullition of the anode with subsequent condensation on the negative ions, causing them to move more slowly. This slow movement would react on the drop in potential at the anode, making it greater.

On the other hand where there are negative ions formed with greater ease, as with NaCl, there would be less ebullition and less condensation on the negative ions. This would account for the smaller drop at the anode, for a greater negative discharge to a surrounding cylinder, and for the disappearance of the magnetic effect if, as would appear, this effect is due to the greater velocity of the positive ions.

It has also been found that there is less condensation on the negative ions when in a vacuum and that there is a corresponding increase in their velocity.[2] This again would explain the smaller drop in potential at the anode, the greater discharge from the arc, and the disappearance of the effect which we have been studying.

But on the other hand it would not explain why the effect does not appear in a vacuum in the opposite direction. Therefore, we can not as yet consider this as a complete explanation.

*Summary.* — If two carbon pencils are placed on the opposite sides of an arc, the potential difference between them is changed when the arc is deflected toward one of them by means of a magnetic field. This change may be as high as four volts. The effect can not be produced by blowing the arc to one side, nor by moving the carbon pencils to one side of the arc. Rapidly reversing the field and with each reversal reversing the connection to the voltmeter does not diminish the effect.

The effect decreases as the strength of the field is diminished. Changes in the current through the arc do not appreciably influence it. It is, however, increased by lengthening the arc.

The effect is smaller with cored than with solid carbons. It decreases and in some cases entirely disappears when salts are

[1] PHYS. REV., 15, 346.
[2] PHYS. REV., 15, 349.

placed in the arc, and it could not be detected with an arc between metals. An attempt was made to observe it with graphite terminals, but it was impossible to maintain an arc between such terminals when in a magnetic field.

The effect decreases as the pressure on the surrounding gas is diminished and with a pressure of approximately 1 cm. of mercury it disappears.

The common equation for the Hall effect does not apply in this case, since there is here a motion of the ions transverse to the direction of the impressed E.M.F. It is possible, however, to explain almost all of the observations by assuming that the positive ions move more rapidly than the negative ones.

COLGATE UNIVERSITY,
December, 1904.

# APPARATUS TO ILLUSTRATE THE PRESSURE OF SOUND WAVES.

By R. W. Wood.

AS is well known the radiometer of Crookes was originally designed to demonstrate the Maxwellian pressure of radiation, and though it was a failure from this point of view, it has become a classical piece of apparatus for showing the reaction pressure of the gas molecule.

The renewed interest in the long sought for pressure which has come up since the beautiful experiments of Nichols and Hull in America, and of Lebedew in Russia, together with the recently published method of demonstrating the tangential component of the pressure by Professor Poynting, makes any piece of apparatus capable of showing wave pressure of considerable value in the lecture room.

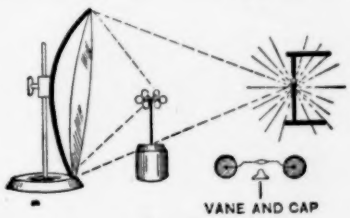

Fig. 1.

While experimenting recently with spark images at the focus of a concave spherical mirror, I was struck with the large amount of mechanical force exerted by the sound waves. These waves could be distinctly felt when the finger was held at the focus, and it occurred to me that a mill-wheel or radiometer driven by these waves would be useful for purposes of demonstration in treating of radiation pressure.

A small wheel was made of aluminum foil and provided with six vanes of thin mica, the whole thing being built up in a few minutes with sealing wax. The bearing was turned out of a small bit of aluminum rod, but the better plan would have been to use the cup bearing of a small compass. The whole was poised on the point of a needle mounted vertically on a small wooden stand.

The wheel spun with great velocity when the sound waves were concentrated on one side by means of the reflector, and by moving the mirror a trifle the motion could be stopped and reversed. I have found that the best plan is to use the discharge of a large coil driven by an alternating current of 110 volts, suitable resistance being included in the circuit. A large Leyden jar is put in parallel with the secondary, and the torrent of very noisy sparks passed between suitable electrodes. The best form of mirror to use is that commonly employed to demonstrate the reflection of radiant heat, a pair of which are to be found in almost every physical laboratory.

The arrangement of the spark, mirror and mill-wheel is shown in the accompanying figure.

# PROCEEDINGS

OF THE

# AMERICAN PHYSICAL SOCIETY.

## MINUTES OF THE TWENTY-SIXTH MEETING.

THE annual meeting of the Physical Society was held in the Morgan Laboratory of Physics, University of Pennsylvania, Philadelphia, in joint session with Section B of the American Association for the Advancement of Science, on Friday, December 30, 1904. President Webster of the Physical Society presided.

The Society first proceeded to the election of officers, Messrs. A. P. Wills and F. A. Waterman being designated by the president to act as tellers. The result of the ballot was the election of the following officers and members of the Council for the year 1905:

*President*, Carl Barus; *Vice-President*, D. B. Brace; *Secretary*, Ernest Merritt; *Treasurer*, Wm. Hallock; *Members of the Council*, A. G. Webster and J. S. Ames.

The following papers were presented:

1. New Series in the Spectra of Mg, Zn, and Cd. F. A. SAUNDERS.

2. A $PQ$ Plane for Thermodynamic Cyclic Analysis. HARVEY N. DAVIS.

3. Maxima and Minima of Atmospheric Nucleation Near to Winter and Summer Solstices. C. BARUS.

4. Objective Method for Showing the Distribution of Nuclei Produced by X-Rays or Other Radiation. C. BARUS.

5. Experiments on Resonance in Wireless Telegraph Circuits. G. W. PIERCE.

6. Gas Spectra in the Region of Extremely Short Wave Lengths. THEODORE LYMAN.

7. A Measurement of the Thomson Thermoelectric Effect in Iron.[1] EDWIN H. HALL.

8. Recent Experiments and the Theories of Ether Drift.[2] D. B. BRACE.

[1] Read before the session of Section B held on December 28, 1904.

[2] Read before the session of Section B held on December 29, 1904.

9. The Luminescence of Sidot Blende. EDWARD L. NICOLS and ERNEST MERRITT.

10. New Phenomena of Fluorescence. HARRY W. MORSE.

11. Infra-red Emission Spectra of Metals. (Read by title.) W. W. COBLENTZ.

12. The Tuning of Thermoelectric Receivers for Electric Waves. (Read by title.) A. D. COLE.

13. Note on the Use of Sensitive Quadrant Electrometers.[1] J. C. MCLENNAN.

14. On the Decay of the Excited Radioactivity from Natural Causes.[1] MISS L. B. JOHNSON.

15. On the Secondary Radiation Excited in Metals by the Gamma Rays from Radium.[1] H. F. DAWES.

16. The Ratio of Electrical to Gravitational Forces. BERGEN DAVIS.

17. A Contribution to the Theory of Thermo-electromotive Force. C. B. THWING.

18. The Capacity and Resistance of Aluminum Anode Films. C. MCC. GORDON.

19. The Specific Heat of Air at Low Temperatures. J. S. SHEARER and R. C. FENNER.

20. Coefficients of Expansion of Solids at Low Temperatures. J. S. SHEARER.

21. The Proper Distribution of Mass in the Kater Pendulum. H. C. RICHARDS.

22. The Nodes of a Bar Vibrating Laterally. H. C. RICHARDS.

23. A Study of the Ring Pendulum Method for the Absolute Determination of $g$. (Read by title.) C. E. MENDENHALL.

24. The Magnetic Susceptibility of Water. A. P. WILLS.

25. Note on the Nature of Electrical Conduction in Metals. C. B. THWING.

26. The Relation b tween Electrical Conductivity and Gas Consumption in the Ordinary Gas Flame. F. L. TUFTS.

27. The Elimination of Gas Action in Experiments on Light Pressure. G. F. HULL.

28. On the Present Shrinkage of the Earth's Magnetism. (Read by Title.) L. A. BAUER.

29. The Evolution of Hydrogen from the Cathode in Gases and its Association with Cathode Rays.[2] C. A. SKINNER.

ERNEST MERRITT, *Secretary.*

ANNOUNCEMENTS.

At the Philadelphia meeting of the Council of the Physical Society it was decided to hold the February meeting of the Society in New York

[1] Read in abstract by the Secretary.

[2] Read before the session of Section B held on December 29, 1904.

City, on Saturday, February 25, 1905, and the spring meeting in Chicago, on Saturday, April 22, 1905.

The vacancies in the Council caused by the election of Carl Barus and D. B. Brace as President and Vice-president, respectively, were filled by the election of Francis E. Nipher and Percival Lewis.

## New Series in the Arc Spectra of Magnesium, Zinc and Cadmium.[1]

By F. A. Saunders.

A FAIRLY complete set of photographs of the arc spectra of magnesium, zinc and cadmium has been taken by the writer: from a brief examination of them, the following preliminary results are presented.

Two series of single lines, running to a common end, have already been found in the spectrum of magnesium; one by Rydberg, the other by Fowler and Shaw. Similar series might well be looked for in the spectra of zinc and cadmium. The spectrum of zinc shows four lines (5182, 4299, 3966, 3799), all somewhat hazy and broadened toward the red, which form such a series, and three lines (6363, 4630 and 4114, with the possible addition of a doubtful line near 3880), which seem to be members of a companion series, running toward the same end. The lines of the second set are still more broadened toward the red and decrease in intensity more rapidly that those of the first. To the first set might be added a line near 7257, though its admission to the series is doubtful, as it seems to prevent the series from satisfying any of the simpler 3-constant formulæ.

The spectrum of cadmium shows two sets which are similar to those of zinc, line for line. They are, for the first set, 5154, 4307, 3982, 3819, 3723, with the possible addition of a line near 7079; for the second group, 6439, 4662, 4141 and 3905. That similar groups exist in the spectrum of mercury seems likely, though their arrangement is less obvious.

On the short wave-length side of four of the members of the first subordinate series of magnesium there are faint triplets (the two last incompletely observed) of nearly the same separations, which bear much the same relation to the strong series as Lenard's new series in the sodium spectrum bears to the subordinate series of that element. Their wave-lengths cannot easily be determined with any precision, as they lie, with one exception, in regions where they are confused with other lines. Preliminary measurements gave 3731, 3051, 2833 and 2729 as the wave-

[1] Abstract of a paper presented at the Philadelphia meeting of the Physical Society, December 30, 1904.

lengths of the first lines of these triplets. They seem to form a third subordinate series of triplets, running to the same end as the other two.

A new line in the magnesium spectrum at 6319, being the only one of considerable intensity in the visible red, apparently belongs to the series found by Fowler and Shaw, though its wave-length does not agree with the value (6674) predicted by them. This is of especial interest, as it emphasizes the difficulties that are to be met with in evolving suitable formulæ to represent these new series.

## A $PQ$ PLANE FOR THERMODYNAMIC CYCLIC ANALYSIS.[1]

BY HARVEY N. DAVIS.

THE object of this paper is to develop a generally applicable method (largely graphical) for the study of the properties of thermodynamic heat-engine cycles.

To determine a cyle of a given type, requires, in general, four coordinates, two to fix its position in the $pv$ plane, and two more to give its dimensions. The coordinates of position used are the initial pressure and temperature, and they are regarded, not as variables, but as parameters in the equations in which they appear. The coordinates of dimension which are chosen are the ratio of compression ($P$), and the amount of heat ($Q$) that a unit mass of working substance takes in from outside sources during one passage around a cycle. If values of $Q$ be used as abscissæ, and values of $P$ as ordinates, these results a $PQ$ plane upon which a point represents and completely characterizes a thermodynamic cycle (when its type and the values of the parameters are known) in exactly the same way that a point on the ordinary $pv$ plane represents and completely characterizes a thermodynamic state (when the form of the characteristic equation is known, together with the values of the constants which it involves). Any property $X$ of cycles of the given type can then be studied by means of the lines $X =$ a constant, which correspond to the isothermals or adiabatics on the older plane.

To illustrate the value of the method, there is carried through, in some considerable detail, an application of it to a simple problem connected with gas turbine cycles.

[1] Abstract of a paper presented at the Philadelphia Meeting of the Physical Society, December 30, 1904.

## Experiments on Resonance in Wireless Telegraph Circuits.[1]

By George W. Pierce.

THE paper is a report of some experiments on the conditions for resonance in wireless telegraph circuits with various lengths of antennæ. The circuits employed were of the electromagnetically connected type. The circuits, apparatus, and method of experiment are described in a previous paper.[2] The receiving instrument is a high frequency dynamometer in which a silver disc (3 mm. in diameter) is hung near a small coil (8 mm. in diameter) of about 30 turns of fine wire. The coil is put in series with the condenser circuit (Circuit IV.) at the receiving station. The disc hangs at an angle of 45° with the plane of the coil. Oscillating currents in the coil induce currents in the disc, which is deflected so as to increase the angle between the disc and the coil. By withdrawing the coil, increasing its distance from the suspended disc, the sensitiveness of the instrument may be diminished, giving the instrument an extremely wide range, so that in a single experiment quantities of energy in the ratio of 1 to 20,000 were compared. The deflections of the instrument are proportional to the square of the current in the coil.

With various antennæ, containing from one to eight wires of which the length was varied from 4 to 24 meters, it was found that the capacity of the receiving antennæ (assumed proportional to its length) is related to the resonant capacity of the condenser in the receiving side circuit in a manner expressed approximately by the equation of a hyperbola,

$$(C_3 - a)(C_4 - b) = k \qquad (1)$$

in which $C_3$ and $C_4$ are respectively the capacity of the receiving mast circuit (Circuit III.) and the capacity of the receiving condenser circuit (Circuit IV.) required for resonance. $a$, $b$, and $k$ are constants for constant inductances at the receiving station and constant frequency of waves.

Equation (1) was also derived theoretically, and the empirical constants were found to have the following values:

$$a = \frac{L_4}{(L_3 L_4 - M^2)\omega^2}, \qquad (2)$$

$$b = \frac{L_3}{(L_3 L_4 - M^2)\omega^2}, \qquad (3)$$

[1] Abstract of a paper presented at the Philadelphia meeting of the Physical Society, December 30, 1904.

[2] Pierce, Physical Review, Vol. XIX., No. 3, September, 1904.

$$k = \frac{M^2}{(L_3 L_4 - M^2)^2 \omega^4} \qquad (4)$$

in which $L_3$ and $L_4$ are respectively the self inductances of Circuits III. and IV., $M$ their mutual inductance, and $\omega$ the angular velocity of the incident waves.

The correctness of equations (2), (3) and (4) was tested by measuring the inductances on Rayleigh's and Maxwell's bridges and by photographing the oscillations in the primary and secondary circuits at the sending station. The resonance conditions are often multiple-valued in agreement with theory.

In the case of sharpest resonance obtained the energy received dropped to half for a change of two per cent. in the period of the condenser circuit at the receiving stations.

CAMBRIDGE, MASS.,
December 19, 1904.

## GAS SPECTRA IN THE REGION OF EXTREMELY SHORT WAVE-LENGTHS.[1]

BY THEODORE LYMAN.

THE substance of this paper consists of a detailed description of the apparatus and methods used in determining the wave-length of light in the region between $\lambda = 1800$ and 1050. Special attention is given to the spectrum of hydrogen and to that of air. Some results are also introduced on the absorption of air and on that of fluorite. This paper is a continuation of the notice read to the Society last February by the author, and will form the subject of a Memoir, shortly to be communicated to the American Academy.

## THE LUMINESCENCE OF SIDOT BLENDE.[2]

BY E. L. NICHOLS AND ERNEST MERRITT.

IN the experiments described in this paper the same spectrophotometric methods were employed as had previousiy been used in the study of the fluorescence of solutions and transparent solids.[3] The blende (phosphorescent zinc sulphide) was in the form of a screen, such as is commonly

[1] Abstract of a paper prescnted at the Philadelphia Meeting of the Physical Society, December 30, 1904.

[2] Abstract of paper presented at the Philadelphia meeting of the Physical Society, December 30, 1904.

[3] Nichols and Merritt, PHYSICAL REVIEW, Vol. 18, p. 447; Vol. 19, p. 18; Vol. 19, p. 396.

used in experiments upon the luminescence due to the rays from radium. The experiments thus far completed deal with luminescence excited by light and by Roentgen rays.

The photoluminescence of the screen studied consisted of two well marked bands with maxima lying respectively in the green ($\lambda = 0.507$) and at the extreme edge of the visible spectrum ($\lambda = 0.432$). It is to the green band that the brilliant and long continued phosphorescence of Sidot blende is due. The duration of phosphorescence in the case of the violet band is too short to be measured by our apparatus, and cannot exceed 0.1 sec. The green band is excited most brilliantly by the violet bands of the carbon arc, but can also be faintly excited by the ultra-violet portion of the spectrum of an arc containing iron or zinc. Using two spectrometers in series, so as to obtain a spectrum of great purity, it was found that the green band was still faintly excited by light lying between $\lambda = 0.472\mu$ and $\lambda = 0.497\mu$. The phosphorescence spectrum could readily be measured from $\lambda = 0.454\mu$ to $\lambda = 0.556\mu$. The green band thus shows the same violation of Stokes' law that we have found in all the cases of fluorescence thus far tested.

The violet band mentioned above is excited by the ultraviolet portion of the spectrum of the iron or zinc arc, but not by light capable of passing through glass. This fact removes all difficulty in studying the two bands separately.

Both bands are excited by Roentgen rays; but the excitation of the violet band is in this case far more intense. Certain differences in the spectrum of the violet band when excited by Roentgen rays and ultra violet light lead us to suspect that this band may be complex. There is some evidence also of the existence of a faint red phosphorescence produced by Roentgen rays.

By using a chronograph and a suitable shutter we were able to determine, in the case of the green band, the curves of decadence for different portions of the phosphorescence spectrum and the form of the phosphorescence spectrum at different times after the excitation had ceased. The decay of phosphorescence could be followed in this way in the spectrophotometer, and measurements could be made, for about 10 sec. During this time the position of the maximum did not change by a measurable amount. The decadence appears to be a little more rapid near the ultra edge of the phosphorescence spectrum, but except for the slight change resulting from this fact the form of the phosphorescence spectrum remains unaltered during decadence.

The decadence curves obtained show a rapid fall in the intensity of phosphorescence during the first two seconds, after which the decay is more gradual. Faint phosphorescence can still be seen, but not measured, after several hours. During the first two or three seconds the decadence

curves are almost exactly hyperbolic, while later, during the period of slow dacay, they are more nearly of the exponential type. A slight modification of the theory proposed by Wiedemann and Schmidt [1] makes it possible to predict the form of the decadence curves, and leads to conclusions that are in agreement with the observed results.

## NEW PHENOMENA OF FLUORESCENCE.[2]

BY HARRY W. MORSE.

BESIDE the broad blue band and the narrower orange one usually noticed in the fluorescent spectrum of fluorspar, and produced by arc, spark or sunlight, the spark between certain metals produces narrow bands and *sharp lines* in certain varieties of the mineral.

Photographs of the fluorescent spectra produced by the following metals have been examined: aluminium, cadmium, iron, lead, magnesium, mercury, tin, zinc. About 125 lines and narrow bands have been measured.

The following points of interest have been observed:

1. The fluorescent spectrum varies from metal to metal.
2. Some lines and bands are common to several metals.
3. Some of the lines observed are at least as sharp as the comparison lines of the spark in air.
4. The light exciting these lines lies between $\lambda$ 3000 and $\lambda$ 2000, since it is cut off nearly completely by microscopic cover glass, but not by quartz plates several millimeters in thickness.
5. Removal of the air lines by the introduction of inductance into the secondary circuit cuts out a large number of the fluorescent lines. A considerable and fundamental extension of our present theory of fluorescence is evidently necessary to include these new facts.

JEFFERSON PHYSICAL LABORATORY, HARVARD UNIVERSITY,
December 18, 1904.

## INFRA-RED EMISSION SPECTRA OF METALS.[3]

BY W. W. COBLENTZ.

PREVIOUS work on this subject is confined to the observations of Julius, and of Paschen, who, by means of the Bunsen flame, or by passing the gases through a solenoid of red-hot platinum wire, found that

[1] Wiedemann's Annalen, Vol. 56, p. 247.

[2] Abstract of a paper presented at the Philadelphia Meeting of the Physical Society, December 30, 1904.

[3] Abstract of a paper presented at the Philadelphia meeting of the Physical Society, December 30, 1904.

the gases of the nonmetals have characteristic, that is, discontinuous spectra. Snow mapped the infrared emission lines of the alkali metals, using for the purpose a glass prism and the chlorides of the metals in the carbon arc. His observations extend to about $2\mu$ and show that there are no strong lines beyond the wave-length $1.5\mu$. Whether this was due to the opacity of the prism was not determined.

The present investigation deals with the question of the distribution of the emission lines in the infra-red spectrum. To this end, the spark spectra of the metals was tried but no emission lines could be detected. The arc between metal electrodes, *e. g.*, Fe and Zn was then tried, but without success because of the incandescent oxides which gave a continuous "black body" spectrum. The only exception was the vapor of copper which had but a slight "black body" radiation. It gave *no* emission lines.

The chloride of Na, Li and K were then examined in the carbon arc using hollow carbon electrodes filled with the salt. The emission lines, found by Snow in the region of $.8\mu$ and $1.2\mu$ were easily detected, but beyond this region to $1.8\mu$ *no* lines could be detected. From 2 to $3\mu$ there is a week continuous spectrum perhaps due to the oxides or air.

At $4.53\mu$ there is an emission band which is quite strong for Na vapor and weaker for that of K and Li. It does not occur in the spectrum of the vapor of the pure carbon arc. Whether this is the atmospheric $CO_2$ band is not known; It would be of interest to know why it appears only in the carbon arc with metallic vapors; it is not found in the arc with copper electrodes. Beyond $5\mu$ the vapors do not show any radiation above that which comes from the box which enclosed the arc. The carbon arc has *no* emission lines beyond $1.2\mu$.

There measurements were made with a mirror spectrometer, a rock salt prism and a radiometer, having a half period of 12 seconds. The chief difficulty in such work is to prevent the "black body" radiation of the electrodes from entering the spectrometer. This was prevented by using an arc about 2 cm. long, and an iron shield with an opening 1 cm. long through which the radiation from the vapors could pass into the spectrometer slit. Any slight trace of the radiation from the terminals was sufficient to give very large deflections, or even to throw the deflection entirely off the scale.

The important results obtained are that the metals examined have *no* emission lines beyond the region investigated by Snow; at least none of measurable intensity. This is entirely unexpected, for in the region from 1.2 to $2.5\mu$ where the "black body" radiation is strongest, at high temperatures, one would expect the emission bands, if there be any, to be just as intense, if not more so than those found by Snow. Pflüger's recent measurements of the distribution of the energy in the ultra-violet

spark spectra of the metals, using a thermopile, show that there is a strong maximum in the ultra-violet, and a weaker one at the end of the red. It is difficult to conceive of an "electrical temperature" (Stark) of some 12,000° to 14,000° (Pflüger) for the vapor between electrodes which have a temperature of about 4,000°. On the other hand, it is easier to think of the vapors having a "pure thermal temperature" the maximum of which lies at the end of the red, which corresponds to a temperature of some 4,000° Abs. The emission band of air at 4.52$\mu$, if it be due to air (*i. e.*, $CO_2$), indicates a temperature of about 4,000° abs. The ultra-violet maximum, as found by means of the thermopile, corresponds somewhat to a region where the spark lines occur in the greatest number, and from existing knowledge may just as likely de due to the nature of the oscillations on the spark as found in King's most recent work on copper, and Crew's work on zinc.

An investigation of the energy from a vacuum tube, using different gases, is in progress, hoping to gain light on this subject. In conclusion it might be added that the mercury arc (Aron's lamp) has no important lines beyond 1.3$\mu$, except in the region of 5$\mu$ where Drew found a large band for a vacuum tube containing air. Runge and Paschen found similar results for Clèveate gas (Helium), in which no strong lines occur beyond 2$\mu$. In all the cases cited the lines in the infra-red region up to 2$\mu$ belong to those of the visible spectrum, as predicted (through wide extrapolation, hence not always in very close agreement with observed lines) by the convergent, "spectral series," formulæ, and as a whole, the data at hand show no lines beyond this point.

PHYSICAL LABORATORY, CORNELL UNIVERSITY,
December 15, 1904.

## CONTRIBUTION TO THE THEORY OF THERMOELECTROMOTIVE FORCE.[1]

BY C. B. THWING.

IN a paper read at the September meeting of the Am. Phys. Society I showed that a relation exists between the rate of variation of the expansion coefficients of the metals compared to lead and their thermo-electromotive force compared with the same metal. J. Traube (Zeits. Vereines Deutsch. Ing., 47, pp. 1189, Aug., 1903) has shown that Van der Waals' equation may be applied to calculate the internal pressures of metals from successive values of their expansion coefficients.

I have calculated the pressures at successive temperatures of the metals and combining the pressures with the known increase in volume have

[1] Abstract of a paper presented at the Philadelphia meeting of the Physical Society, December 30, 1904.

calculated the potential, $V = p\, dv/dt$ for two temperatures. Then the numbers $dV/dt - dV/dt_{(Pb)}$ are proportional to the thermoelectromotive potential differences for the same metals.

## The Proper Distribution of Mass in the Kater Pendulum.[1]

By Horace C. Richards.

IN experimenting with the Kater pendulum it is sometimes found impossible to adjust the slider so as to obtain equal periods. An alteration in the distribution of mass must then be made. Practically, with a given pendulum we may alter either the bob or the slider. A discussion of the variations may be summarized as follows:

1. In a convertible pendulum provided with a slider between the knife-edges, the slider *may*, but only in exceptional cases *will* increase the period. It then acts more by virtue of its inertia than of its weight.

2. Usually the slider dimishes the period; therefore the pendulum with the slider removed should have an equivalent length greater than the distance between the knife-edges.

3. For a given pendulum the mass of the slider must be between definite limits in order that it may be possible to adjust to equality the periods of oscillation about the two knife-edges.

4. Conversely, with a given slider the mass of the bob must be within definite limits. However, in some cases it will not be possible to obtain equal periods by altering the bob alone.

## The Nodes of a Bar Vibrating Laterally.

By Horace C. Richards.

THE theory of the free lateral vibrations of a thin elastic bar was first given completely in 1847 by Seebeck, who determined the periods for the six classes of vibrations and located the nodes, points of inflection, etc. Later writers, including Donkin and Rayleigh, have drawn largely from his paper and quoted his results. An examination of the values given for the nodal points shows some inconsistencies, indicating that those for the "clamped-free" bar were in error, a conclusion confirmed by careful drawings of the curves. The nodal points were therefore calculated anew for the first few tones by the method given by Rayleigh, and also general expressions for the higher tones. A comparison is made between the position of corresponding nodes with different

[1] Abstract of a paper presented at the Philadelphia meeting of the Physical Society, December 30, 1904.

terminal conditions, and it is shown that the differences may be expressed in the following general statement: changing one end of a bar from free to clamped shifts all the odd-numbered nodes (counting from that end) towards that end, and all the even-numbered nodes away from the end. Changing both ends from free to clamped has the same effect counting from each end to the center of the bar.

## A STUDY OF THE RING-PENDULUM METHOD FOR THE ABSOLUTE DETERMINATION OF "$g$."[1]

BY C. E. MENDENHALL.

THE method in question was first suggested by Dr. T. C. Mendenhall, and this paper, which will be published in full by the National Academy of Sciences, contains the results of the first careful test of its possibilities. Reference must be made to the complete paper for the details of manipulation and the construction of glass-hard steel rings of sufficiently accurate figure for the purpose. Two such rings gave independently the following values for "$g$" at Madison, Wisconsin.

$$(1)\ g = 980.526$$
$$(2)\ g = 980.511$$

which is considered a very satisfactory agreement.

The approximate dimensions of the rings are:

External diameter = $28.8^{cm}$. } for a complete period
Internal diameter = $16.6^{cm}$. } of about $1^{s}.003$.
Thickness = $1.5^{cm}$.

One of the rings proved to be of considerably less uniform density than the other; and undoubtedly a considerably closer agreement would be found between rings of equally homogeneous texture. The advantages of the method are pointed out to be (1) rigidity and permanence of form of the pendulums (2) size and consequent ease of making accurate length measurements.

## ON THE PRESENT SHRINKAGE OF THE EARTH'S MAGNETISM.[2]

BY L. A. BAUER.

A TABULATION of the values of the earth's magnetic moment as derived by various analysts for the period 1843 to 1885 show a

[1] Abstract of a paper presented at the Philadelphia Meeting of the Physical Society, December 30, 1904.

[2] Abstract of a paper presented at the Philadelphia Meeting of the Physical Society, December 30, 1904.

diminution for the interval of 40 years of about 1.6 per cent. Computations of the earth's total magnetic energy made by the author for the same period proved that the shrinkage in the portion equivalent to a uniform magnetization was not compensated by a corresponding increase in the remaining portion of the earth's magnetism. As all of these results, however, were obtained on the basis of the distribution of the earth's magnetism as portrayed by the existing magnetic charts, which owing to paucity of data especially in the oceanic areas, are more or less inaccurate, the deductions regarding a possible shrinkage of the earth's magnetism could not be accepted as absolutely conclusive.

In the present paper the results derived depend on actual observations between 1890 and 1900 at a series of land stations fairly uniformly distributed over the earth and lying between the parallels 60° N. and 60° S., the same number of stations having been selected in each magnetic hemisphere.

From the mathematical analysis of the secular variation of the earth's magnetism between 1890 and 1900, it is found that the observed secular variation in the uniform portion of the earth's magnetism is to be ascribed to systems of magnetic or electric forces two of which are as follows:

1. A system situated *below* the earth's surface, having a potential and acting in opposition to the earth's momentary field, hence demagnetizing it. The magnetic axes of this system of forces besides being reversed with reference to that of the earth's momentary field is also displaced towards the west so that its effect is to cause the north magnetic pole to move westward and southward and to diminish the earth's magnetic moment at the rate of about one twenty-two-hundredth part annually.

2. A system situated *above* the earth's surface, also having a potential, having its magnetic axis directed nearly transverse to the earth's axis of rotation. This system plays a most important part in the secular variation of the magnetic declination.

The annual rate of shrinkage of the earth's magnetic moment for the period 1890 to 1900 is practically the same as that deduced on the basis of the magnetic charts for 1843 to 1885, adopting the mean of the two ratios we find that the earth's magnetic moment has been shrinking since 1840 at the rate of about one twenty-four-hundredth part of its prevailing value annually. At this rate the earth's magnetic moment will have dwindled to one half its present value in 1,600 years. However, we have not yet sufficient grounds for such predictions. All we know is that a shrinking is occurring now and the indications are that this shrinkage will be found to have been prevailing as far back as our available data extend.

It is thus found *that the secular variation of the earth's magnetism results not only from a change in the direction of the axis of magnetization but also from a change in the intensity of magnetization.*

## THE CAPACITY AND RESISTANCE OF ALUMINUM ANODE FILMS.[1]

BY C. MCC. GORDON.

THE films formed on aluminum when used as anode, have been the subject of numerous investigations in recent years. The thickness, the chemical nature, the resistance, the specific inductive capacity of these films, the voltage at which they break down and the variations in their properties with the nature of the electrolyte are questions concerning which the evidence is varied and in many cases apparently contradictory.

Most of the measurements heretofore made have been carried out with the use of direct currents. The use of alternating currents with the wheatstone bridge, possesses advantages over the direct current methods in that the capacity, resistance of the film, and the resistance of the electrolyte can be independently measured at the same time.

Using this latter method experiments which I have recently made show :

1. The capacity of the film is inversely proportional to the volts used to form the film. It is to be inferred from this that the thickness of the film is proportional to the volts applied in its formation.

2. For water solutions, the capacity of the film varies but slightly with the nature of the electrolyte, provided of course the electrolyte is such as to form a film at all noticeable. This is not in accord with the theory recently advanced by Mott, *i. e.*, that the films would be thinner the greater the valence of the acid radical of the electrolyte.

3. The capacity of the films formed in molten electrolytes is much less than those of water solutions.

4. The capacity of the films in molten electrolytes increases rapidly after the voltage applied to form the film is taken away, indicating a rapid dissolving of the film in molten electrolytes.

5. The resistance of the films is much less than is given by many investigators.

[1] Abstract of a paper presented at the Philadelphia Meeting of the Physical Society, December 30, 1904.